**CONTINUITY** **Math21a, O. Knill**

HOMEWORK: 11.1: 16,22,31-36, 11.2: 30,36

CONTINUITY: A function $f(x,y)$ with domain $R$ is is **continuous at a point** $(a,b) \in R$ if if $f(x,y) \to f(a,b)$ whenever $(x,y) \to (a,b)$.

The function $f$ is **continuous on** $R$, if $f$ is continuous for every point $(a,b)$ on $R$.

EXAMPLES.

a) $f(x,y) = x^2 + y^4 + xy + \sin(y + \sin\sin\sin\sin(x)^2$ is continuous on the entire plane. It is built up from functions which are continuous using addition, multiplication and composition.

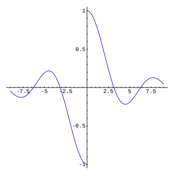
b) $f(x,y) = 1/x^2 + y^2$ is continuous everywhere except at the origin, where it is not defined.

c) $f(x,y) = y + \sin(x)/|x|$ is continuous away from $x = 0$. At every point $(0,y)$ it is discontinuous. $f(1/n,y) \to y+1$ and $f(1/n,y) \to y-1$ for $n \to \infty$.

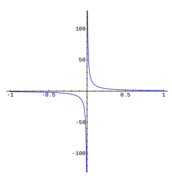
d) $f(x,y) = \sin(1/(x+y))$ is continuous except on the line $x + y = 0$.

THREE SOURCES FOR DISCONTINUITY. Roughly speaking there are three reasons, why a function can be discontinuous: it can **jump**, it can **diverge to infinity**, or it can **oscillate**. The prototypes in one dimensions are

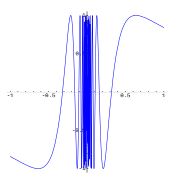
JUMP $f(x) = \sin(x)/|x|$



DIVERGE $g(x) = 1/x$



OSCILLATE $h(x) = \sin(1/x)$



One can have mixtures of these three things of course like the function $f(x)+h(x)$, which jumps and a oscillatory problem at $x = 0$.

HOW DO WE CHECK FOR CONTINUITY? There are two handy tools to check whether a function $f(x,y)$ is continuous at a point:

1) Use polar coordinates with coordinate center at the point.

2) Restrict the function to one dimensional curves and check continuity there.

EXAMPLE I. $f(x,y) = \frac{\sin(x^2+y^2)}{x^2+y^2}$.

EXAMPLE II. $f(x,y) = \frac{x^2-y^2}{x^2+y^2}$.

EXAMPLE III. $f(x,y) = \frac{x^2 y}{x^4+y^2}$.

EXAMPLE IV. $f(x,y) = \frac{xy^2+y^3}{x^2+y^2}$.

EXAMPLE V. $f(x,y) = \frac{\sin(1/(x^2+y^2))}{\sqrt{x^2+y^2}}$.