

Here are six functions whose graphs we ought to be able to visualize in space:

1 $z = f(x, y) = 6 - 3x - 2y$

2 $z = f(x, y) = x^2 + y^2$

3 $z = f(x, y) = x^2 - y^2$

4 $z = f(x, y) = x^2 + y + 1$

5 $z = f(x, y) = (x - y)^2$

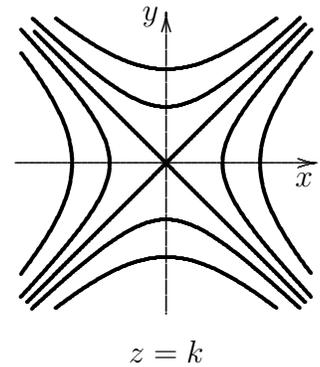
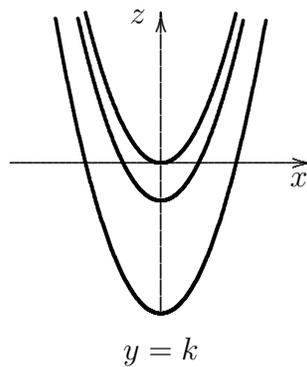
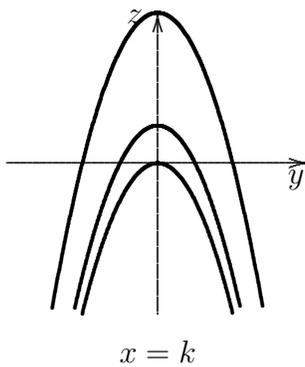
6 $z = f(x, y) = \frac{1}{1 + x^2 + y^2}$

Traces: One way to visualize surfaces that are three-dimensional graphs is to view pieces of them as two-dimensional graphs. If we intersect the graph of $z = f(x, y)$ with a plane (such as $x = k$ or $y = k$), we get a graph in a two-dimensional plane (the kind we're used to). This is called a *trace*. If we intersect the graph of $z = f(x, y)$ with the plane $z = k$, we get *level curves* (not necessarily graphs, although sometimes also called traces).

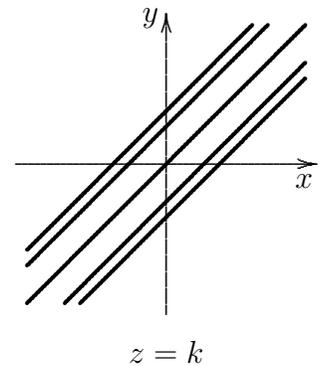
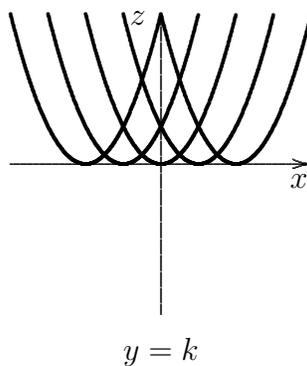
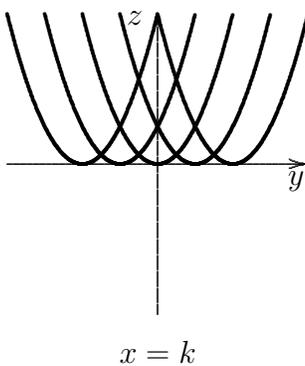
1 Here are traces for two of the surfaces above. Your job is to:

- (a) identify which graph these traces belong with,
- (b) graph traces of the remaining graphs of surfaces, and
- (c) try to visualize the original surface.

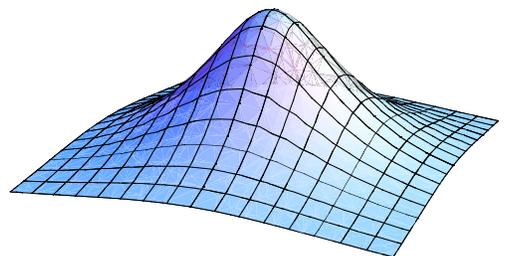
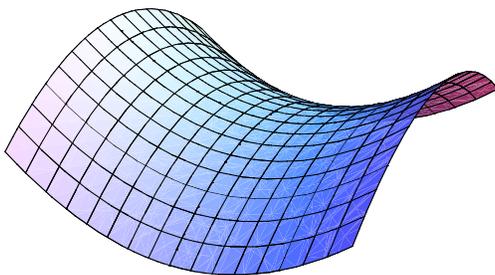
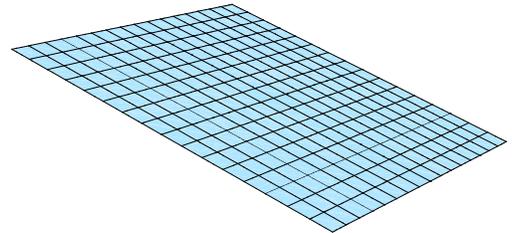
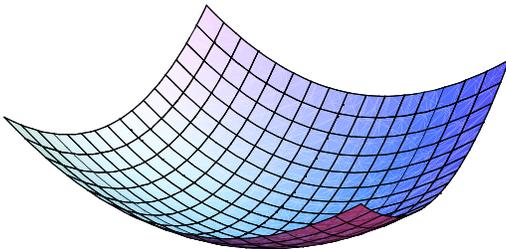
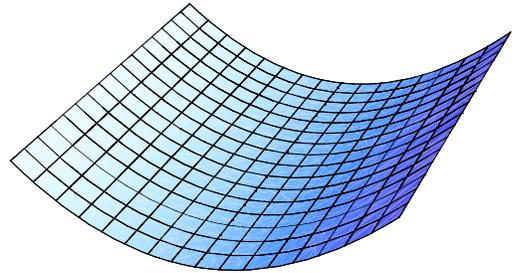
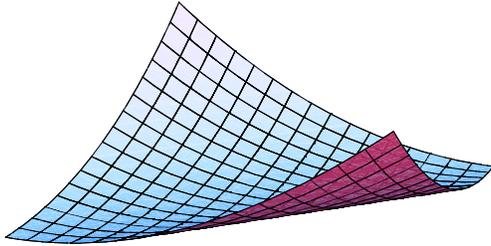
First set of traces (each with $k = 0, \pm 1$, and ± 2 , when possible):



Second set of traces (each with $k = 0, \pm 1$, and ± 2 , when possible):



- 2 Now that you have a good idea of what each of these graphs look like, you should have no problem identifying which of the following (axes-less) graphs go with each equation for the previous page. Your reasoning should involve the graphs of traces you drew as well.



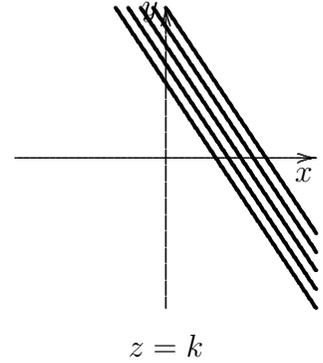
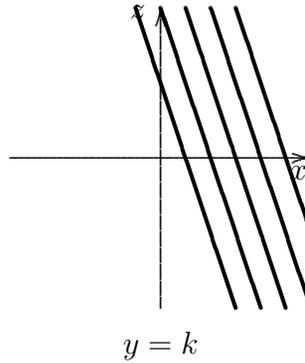
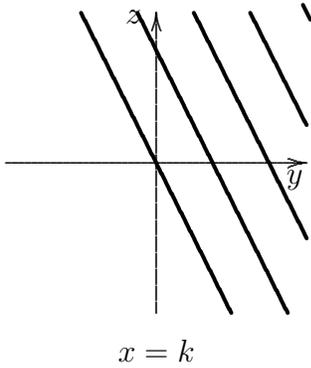
Graphs in Space – Answers and Solutions

For solutions, we'll show the traces and appropriate graph for each of the equations.

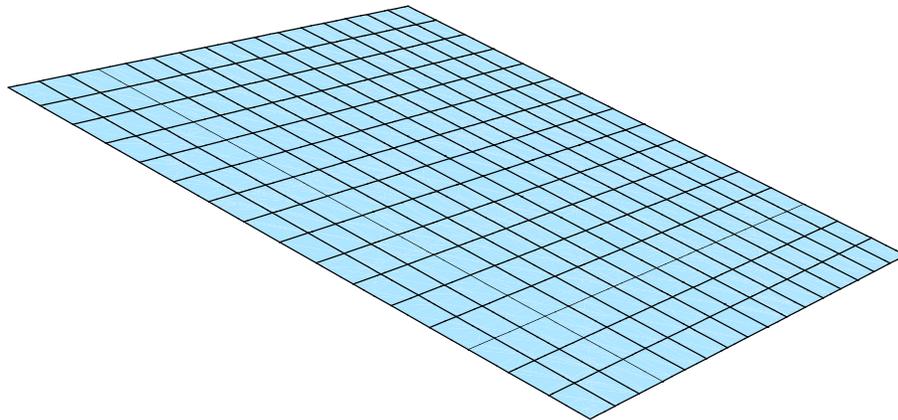
Warning: The graphs are not necessarily all shown from the same perspective! They have been rotated to show the appropriate features, so don't assume that the coordinate axes are positioned in any particular way!

1 $z = f(x, y) = 6 - 3x - 2y$

Here are the traces for $k = -2, -1, 0, 1,$ and 2 :

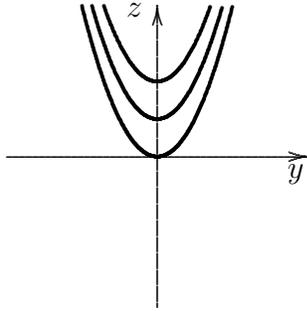


All these traces are lines, which makes sense as the original equation is simply $3x + 2y + z = 6$, a plane!

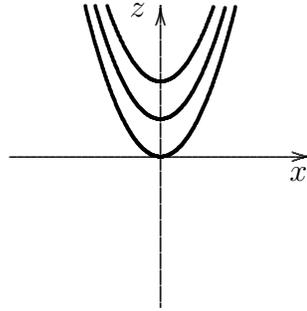


2 $z = f(x, y) = x^2 + y^2$

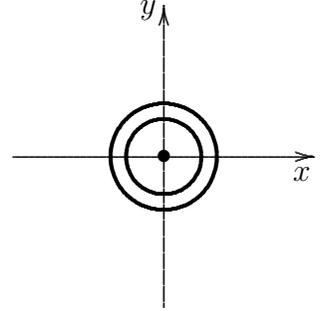
Here are the traces for $k = -2, -1, 0, 1,$ and 2 :



$x = k$

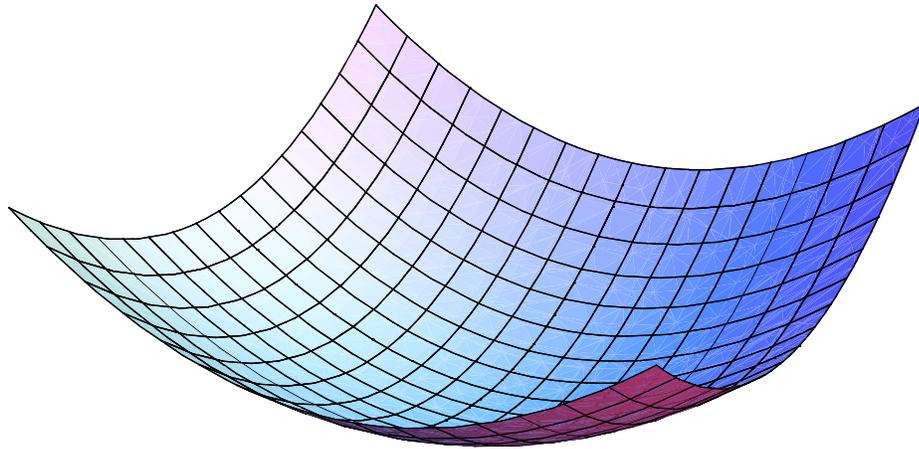


$y = k$



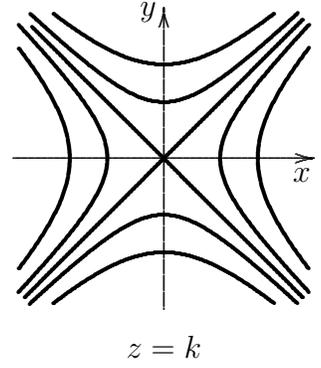
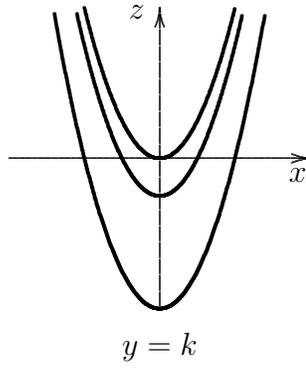
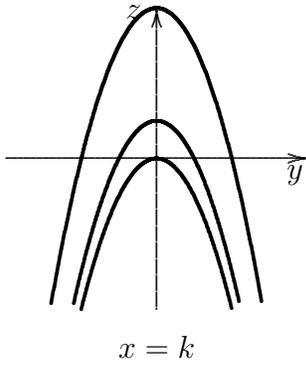
$z = k$

The $z = k$ traces are circles $x^2 + y^2 = k$ of radius \sqrt{k} (so there is no trace at all if $k < 0$).

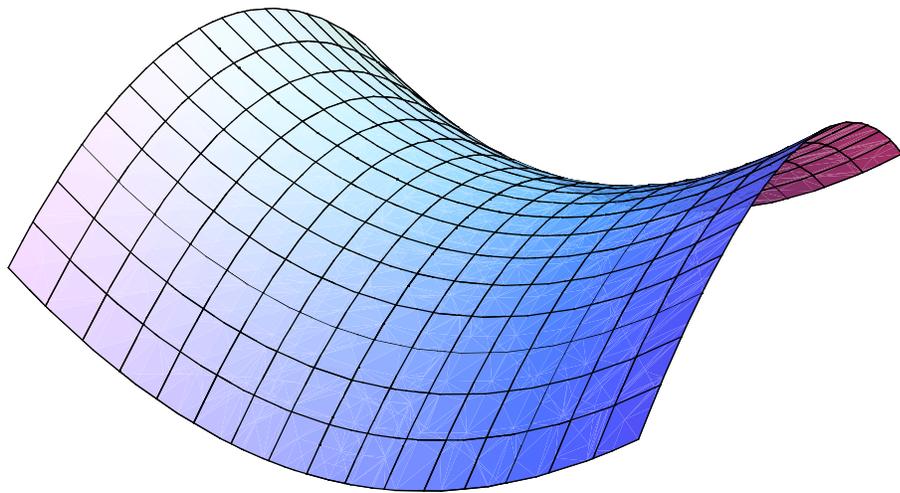


3 $z = f(x, y) = x^2 - y^2$

This is the first set of traces (from the first page of the worksheet):

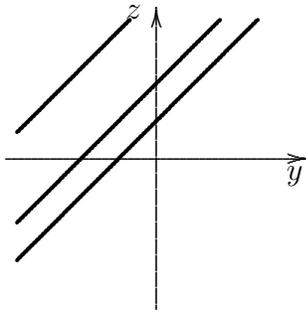


Note the hyperbolas in the $z = k$ traces.

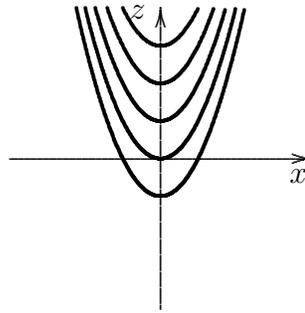


4 $z = f(x, y) = x^2 + y + 1$

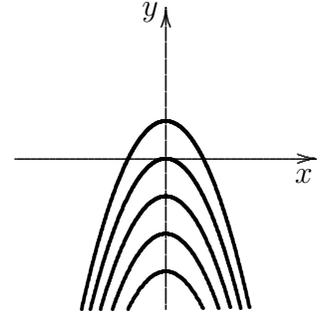
Here are the traces for $k = -2, -1, 0, 1,$ and 2 :



$x = k$

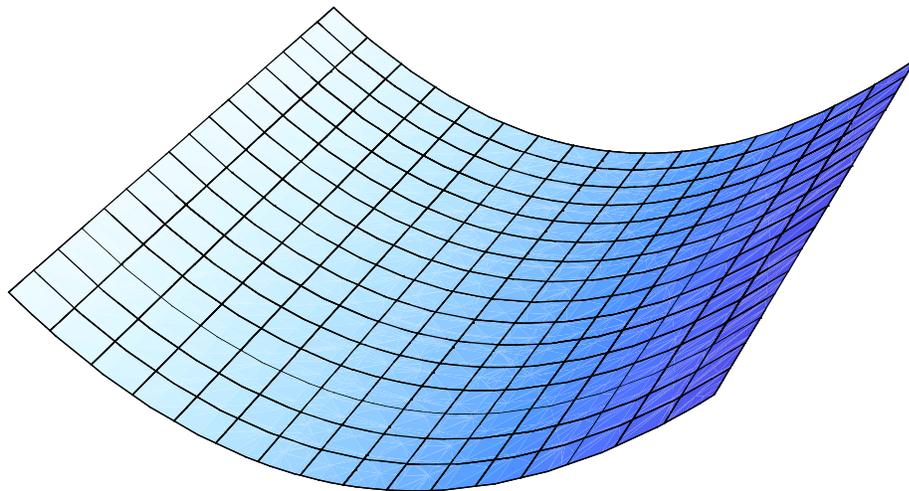


$y = k$



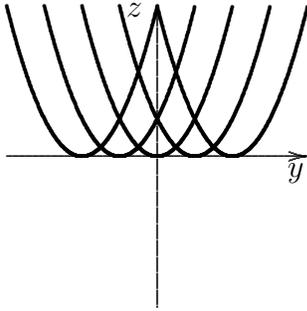
$z = k$

The $z = k$ traces are the lines $x - y = \pm\sqrt{k}$ or $y = x \mp \sqrt{k}$.

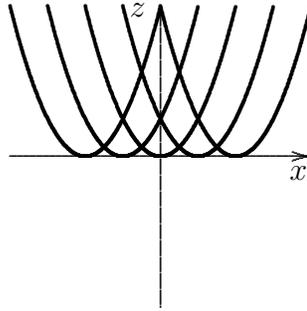


5 $z = f(x, y) = (x - y)^2$

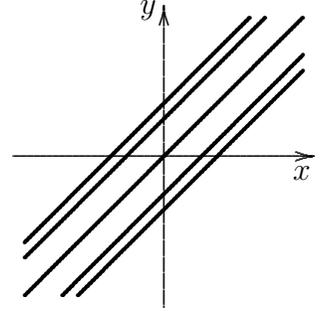
This is the second set of traces (from the first page of the worksheet):



$x = k$

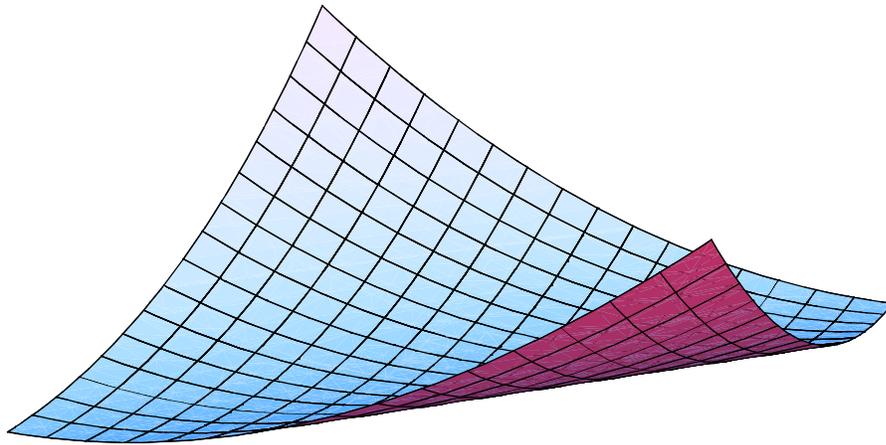


$y = k$



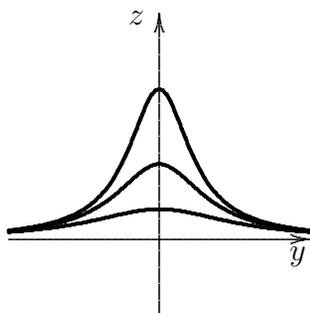
$z = k$

The $z = k$ traces are the lines $x - y = \pm\sqrt{k}$ or $y = x \mp \sqrt{k}$.

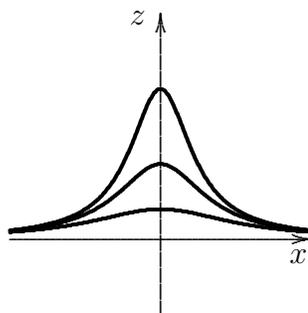


$$\boxed{6} \quad z = f(x, y) = \frac{1}{1 + x^2 + y^2}$$

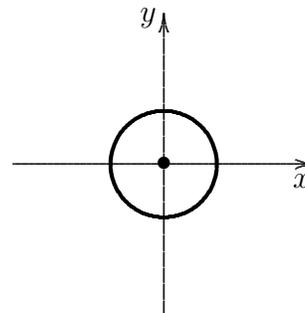
Here are the traces for $k = -2, -1, 0, 1,$ and 2 .



$x = k$



$y = k$



$z = k$

Notice that the $z = k$ traces are only sensible for $k > 0$, so there are only two traces. These are each a circle of radius $\sqrt{1 - \frac{1}{k}}$, so when $k = 1$ this circle has degenerated into a point. The $x = k$ and $y = k$ traces, on the other hand, are the same graphs and don't depend on the sign of k .

