

For each of the following, we'd like four descriptions of the point, surface, or solid:

- (a) An equation in Cartesian coordinates (the usual  $x, y, z$ ).
- (b) An equation in cylindrical coordinates  $(r, \theta, z)$ .
- (c) An equation in spherical coordinates  $(\rho, \theta, \phi)$ .
- (d) A description or graph of the surface or curve.

For each of the following points, surfaces and solids, expand the description to include all four of the above approaches.

**Points:**

1  $(x, y, z) = (1, 2, 3)$

2  $(r, \theta, z) = (3, \frac{\pi}{4}, 3)$

3  $(\rho, \theta, \phi) = (2, \frac{\pi}{3}, \frac{\pi}{6})$

**Surfaces:**

4  $\theta = \frac{\pi}{4}$

5  $x^2 + y^2 + z^2 - 2z = 0$

6  $r = 2 \cos(\theta)$

(This is in cylindrical coordinates. Does it matter?)

7  $\phi = \frac{\pi}{4}$

8  $z^2 - x^2 - y^2 = 1$

9  $\rho = 2 \cos(\phi)$

**Solids:**

10  $3 \leq r \leq 5$

11  $\frac{\pi}{6} < \phi < \frac{\pi}{4}$

12  $\cos(\theta) < r < 1$

13 The solid formed by a sphere of radius 2 centered at the origin with a cylinder removed. Assume the cylinder has radius 1 and is centered on the  $z$ -axis.

## Other Coordinate Systems – Answers and Solutions

### Points:

$$\boxed{1} \quad (r, \theta, z) = (\sqrt{5}, \tan^{-1}(2), 3) \approx$$

$$(\rho, \theta, \phi) = (\sqrt{14}, \tan^{-1}(2), \cos^{-1}(3/\sqrt{14})) \approx$$

$$\boxed{2} \quad (x, y, z) = (3 \cos(\pi/4), 3 \sin(\pi/4), 3) = \left(\frac{3}{\sqrt{2}}, \frac{3}{\sqrt{2}}, 3\right) \approx$$

$$(\rho, \theta, \phi) = (3\sqrt{2}, \frac{\pi}{4}, \frac{\pi}{4})$$

$$\boxed{3} \quad (x, y, z) = (2 \sin(\pi/6) \cos(\pi/3), 2 \sin(\pi/6) \sin(\pi/3), 2 \cos(\pi/6)) = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}, \sqrt{3}\right)$$

$$(r, \theta, z) = (2 \sin(\pi/6), \frac{\pi}{3}, 2 \cos(\pi/6)) = (1, \frac{\pi}{3}, \sqrt{3})$$

### Surfaces:

$$\boxed{4} \quad \text{(a) } y = x \text{ but also } x \geq 0.$$

$$\text{(b) } \theta = \frac{\pi}{4}$$

$$\text{(c) } \theta = \frac{\pi}{4} \text{ (the same in spherical as in cylindrical coordinates)}$$

(d) This is a half-plane over (and under) the part of the line  $y = x$  in the first quadrant (including the origin)

$$\boxed{5} \quad \text{(a) } x^2 + y^2 + (z - 1)^2 = 1$$

$$\text{(b) } r^2 + (z - 1)^2 = 1$$

$$\text{(c) } \rho^2 = 2\rho \cos(\phi) \text{ or simply } \rho = 2 \cos(\phi)$$

(d) This is a sphere of radius 1 centered at  $(x, y, z) = (0, 0, 1)$ .

$$\boxed{6} \quad \text{(a) } (x - 1)^2 + y^2 = 1$$

$$\text{(b) } r = 2 \cos(\theta)$$

$$\text{(c) } \rho \sin(\phi) = 2 \cos(\theta) \text{ or perhaps } \rho = \frac{2 \cos(\theta)}{\sin(\phi)}$$

(d) This is a cylinder of radius 1 centered about the line  $\mathbf{r}(t) = \langle 1, 0, 0 \rangle + t\langle 0, 0, 1 \rangle$ . That is, it is a vertical cylinder above the circle of radius 1 centered in the  $xy$ -plane at the point  $(x, y) = (1, 0)$ .

$$\boxed{7} \quad \text{(a) } z = \sqrt{x^2 + y^2}. \text{ (The top half only of } z^2 = x^2 + y^2 \text{.)}$$

$$\text{(b) } z = r$$

$$\text{(c) } \phi = \frac{\pi}{4}$$

(d) This is the top half of a cone. It opens up, centered along the  $z$ -axis, with “point” at the origin. The trace  $z = k$  for positive  $k$  is a circle of radius  $k$  (centered at the origin of the  $xy$ -plane).

- 8 (a)  $z^2 - x^2 - y^2 = 1$   
 (b)  $z^2 - r^2 = 1$   
 (c)  $\rho = \frac{1}{\sqrt{2 \cos^2(\phi) - 1}}$  or perhaps just  $\rho^2 (2 \cos^2(\phi) - 1) = 1$ .  
 (d) This is a hyperboloid of two sheets.

9 This is the same as Problem 5.

### Solids:

- 10 (a)  $9 < x^2 + y^2 < 25$   
 (b)  $3 < r < 5$   
 (c)  $\frac{3}{\sin(\phi)} < \rho < \frac{5}{\sin(\phi)}$  or simply  $3 < \rho \sin(\phi) < 5$  or simply  
 (d) This is a “solid cylinder” centered around the  $z$ -axis with interior radius 3 and exterior radius 5.
- 11 (a) The “top half” of  $x^2 + y^2 < z^2 < 3(x^2 + y^2)$ ; that is,  $\sqrt{x^2 + y^2} < z < \sqrt{3}\sqrt{x^2 + y^2}$ .  
 (b) The “top half” of  $r^2 < z^2 < 3r^2$ ; that is,  $r < z < \sqrt{3}r$ .  
 (c)  $\frac{\pi}{6} < \phi < \frac{\pi}{4}$   
 (d) This is a “thickened cone” centered around the  $z$ -axis where the exterior angle is  $\pi/4$  and the interior angle is  $\pi/6$ .
- 12 (a) In the  $xy$ -plane, this is the intersection of  $(x - \frac{1}{2})^2 > \frac{1}{4}$  (the points outside the circle of radius  $1/2$  centered at  $(x, y) = (1/2, 0)$ ) and  $x^2 + y^2 < 1$  (the points inside the unit circle centered at the origin). In space, therefore, it is the solid unit cylinder (centered around the  $z$ -axis) with a cylinder removed (the removed cylinder is centered around the line  $x = 1/2, y = 0$  and has radius  $1/2$ ).  
 (b)  $\cos(\theta) < r < 1$ .  
 (c)  $\cos(\theta) < \rho \sin(\phi) < 1$  or  $\frac{\cos(\theta)}{\sin(\phi)} < \rho < \frac{1}{\sin(\phi)}$ .  
 (d) See the answer to (a).
- 13 This is very similar to Example 8 on page 688 of the text, so you might prefer to read that.  
 (a)  $1 \leq x^2 + y^2 \leq 4 - z^2$   
 (b)  $1 \leq r^2 \leq 4 - z^2$  or  $1 \leq r \leq \sqrt{4 - z^2}$   
 (c)  $\sqrt{1 + \rho^2 \cos^2(\phi)} \leq \rho \leq 2$  or  $1 + \rho^2 \cos^2(\phi) \leq \rho^2 \leq 4$ .  
 (d) See the statement. The assumption here is that it is a ball (a “solid” sphere) rather than simply a sphere.