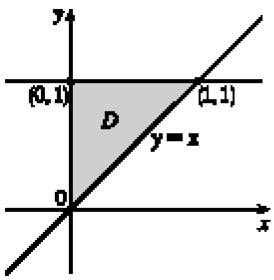
12.3: 20, 30, 40, 44, 46

20.

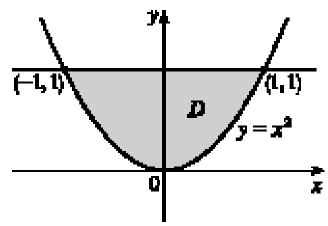


$$V = \int_0^1 \int_x^1 (x^2 + 3y^2)\, dy\, dx$$

$$= \int_0^1 \left[x^2 y + y^3\right]_{y=x}^{y=1} dx = \int_0^1 (x^2 + 1 - 2x^3)\, dx$$

$$= \left[\tfrac{1}{3}x^3 + x - \tfrac{1}{2}x^4\right]_0^1 = \tfrac{5}{6}$$

30. The two planes intersect in the line $y = 1$, $z = 3$, so the region of integration is the plane region enclosed by the parabola $y = x^2$ and the line $y = 1$. We have $2 + y \geq 3y$ for $0 \leq y \leq 1$, so the solid region is bounded above by $z = 2 + y$ and bounded below by $z = 3y$.

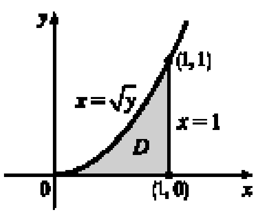


$$V = \int_{-1}^1 \int_{x^2}^1 (2 + y)\, dy\, dx - \int_{-1}^1 \int_{x^2}^1 (3y)\, dy\, dx = \int_{-1}^1 \int_{x^2}^1 (2 + y - 3y)\, dy\, dx$$

$$= \int_{-1}^1 \int_{x^2}^1 (2 - 2y)\, dy\, dx = \int_{-1}^1 \left[2y - y^2\right]_{y=x^2}^{y=1} dx$$

$$= \int_{-1}^1 (1 - 2x^2 + x^4)\, dx = x - \tfrac{2}{3}x^3 + \tfrac{1}{5}x^5\Big]_{-1}^1 = \tfrac{16}{15}$$

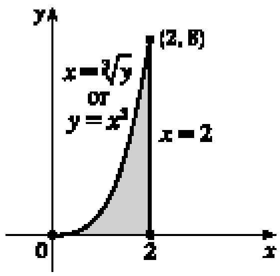
40.



$$\int_0^1 \int_{\sqrt{y}}^1 \sqrt{x^3 + 1}\, dx\, dy = \int_0^1 \int_0^{x^2} \sqrt{x^3 + 1}\, dy\, dx = \int_0^1 \left[\sqrt{x^3 + 1}\, y\right]_{y=0}^{y=x^2} dx$$

$$= \int_0^1 x^2 \sqrt{x^3 + 1}\, dx = \tfrac{2}{9}(x^3 + 1)^{3/2}\Big]_0^1 = \tfrac{2}{9}(2^{3/2} - 1)$$

44.



$$\int_0^8 \int_{\sqrt[3]{y}}^2 e^{x^4}\,dx\,dy = \int_0^2 \int_0^{x^3} e^{x^4}\,dy\,dx$$
$$= \int_0^2 e^{x^4}\left[y\right]_{y=0}^{y=x^3}\,dx = \int_0^2 x^3 e^{x^4}\,dx$$
$$= \tfrac{1}{4}e^{x^4}\Big]_0^2 = \tfrac{1}{4}(e^{16}-1)$$

46. $D = \left\{(x,y) \mid -1 \le x \le 0,\ -1 \le y \le 1+x^2\right\} \cup \left\{(x,y) \mid 0 \le x \le 1, \sqrt{x} \le y \le 1+x^2\right\}$

$\cup \{(x,y) \mid 0 \le x \le 1,\ -1 \le y \le -\sqrt{x}\,\}$,

all type I.

$$\iint_D xy\,dA = \int_{-1}^0 \int_{-1}^{1+x^2} xy\,dy\,dx + \int_0^1 \int_{\sqrt{x}}^{1+x^2} xy\,dy\,dx + \int_0^1 \int_{-1}^{-\sqrt{x}} xy\,dy\,dx$$
$$= \int_{-1}^0 \left[\tfrac{1}{2}xy^2\right]_{y=-1}^{y=1+x^2}\,dx + \int_0^1 \left[\tfrac{1}{2}xy^2\right]_{y=\sqrt{x}}^{y=1+x^2}\,dx + \int_0^1 \left[\tfrac{1}{2}xy^2\right]_{y=-1}^{y=-\sqrt{x}}\,dx$$
$$= \int_{-1}^0 \left(x^3 + \tfrac{1}{2}x^5\right)\,dx + \int_0^1 \tfrac{1}{2}(x^5 + 2x^3 - x^2 + x)\,dx + \int_0^1 \tfrac{1}{2}(x^2 - x)\,dx$$
$$= \left[\tfrac{1}{4}x^4 + \tfrac{1}{12}x^6\right]_{-1}^0 + \tfrac{1}{2}\left[\tfrac{1}{6}x^6 + \tfrac{1}{2}x^4 - \tfrac{1}{3}x^3 + \tfrac{1}{2}x^2\right]_0^1 + \tfrac{1}{2}\left[\tfrac{1}{3}x^3 - \tfrac{1}{2}x^2\right]_0^1 = -\tfrac{1}{3} + \tfrac{5}{12} - \tfrac{1}{12} = 0$$