Homework 4: Three More Problems

1 (a) Which of the following methods will always give you the distance between two parallel lines L_1 and L_2?

(i) The distance is always 0.
(ii) Pick any point P on line L_1 and any point Q on line L_2, and find the distance $|\overrightarrow{PQ}|$.
(iii) Pick any point P on line L_1, and find the distance from P to L_2.
(iv) Pick any point P on line L_1 and any plane $\mathcal{S}$ which contains line L_2 and is parallel to line L_1. Find the distance from P to the plane $\mathcal{S}$.

(b) Which of the following methods will always give you the distance between two non-parallel lines L_1 and L_2?

(i) The distance is always 0.
(ii) Pick any point P on line L_1 and any point Q on line L_2, and find the distance $|\overrightarrow{PQ}|$.
(iii) Pick any point P on line L_1, and find the distance from P to L_2.
(iv) Pick any point P on line L_1 and any plane $\mathcal{S}$ which contains line L_2 and is parallel to line L_1. Find the distance from P to the plane $\mathcal{S}$.

(c) Which of the following methods will always give you the distance between two parallel planes $\mathcal{S}_1$ and $\mathcal{S}_2$?

(i) The distance is always 0.
(ii) Pick any point P on plane $\mathcal{S}_1$ and any point Q on plane $\mathcal{S}_2$, and find the distance $|\overrightarrow{PQ}|$.
(iii) Pick any point P on plane $\mathcal{S}_1$ and any line L in plane $\mathcal{S}_2$, and find the distance from P to L.
(iv) Pick any point P on plane $\mathcal{S}_1$, and find the distance from P to $\mathcal{S}_2$.

(d) Which of the following methods will always give you the distance between two non-parallel planes $\mathcal{S}_1$ and $\mathcal{S}_2$?

(i) The distance is always 0.
(ii) Pick any point P on plane $\mathcal{S}_1$ and any point Q on plane $\mathcal{S}_2$, and find the distance $|\overrightarrow{PQ}|$.
(iii) Pick any point P on plane $\mathcal{S}_1$ and any line L in plane $\mathcal{S}_2$, and find the distance from P to L.
(iv) Pick any point P on plane $\mathcal{S}_1$, and find the distance from P to $\mathcal{S}_2$.

2 In each part, first decide whether the lines L_1 and L_2 are parallel. (Explain how you know.) Then, use the method you chose in the appropriate part of Problem 1 to find the distance between L_1 and L_2.

(a) L_1 is defined by the parametric vector equation $\mathbf{r}_1(t) = \langle 1,3,5\rangle + t\langle 2,0,1\rangle$, and L_2 is defined by the parametric vector equation $\mathbf{r}_2(t) = \langle 2,7,4\rangle + t\langle 0,-1,1\rangle$.

(b) L_1 is defined parametrically by $x = 1+2t$, $y = 3-t$, and $z = 7+3t$. L_2 passes through the point $(5,1,13)$ and is parallel to $\langle 1,3,0\rangle$.

(c) L_1 is defined by the symmetric equations $x-2 = \frac{y+1}{2} = \frac{3-z}{2}$, and L_2 is defined parametrically by $x = 5-t$, $y = -2t$, and $z = 1+2t$.

3 In each part, first decide whether the planes $\mathcal{S}_1$ and $\mathcal{S}_2$ are parallel. (Explain how you know.) Then, use the method you chose in the appropriate part of Problem 1 to find the distance between $\mathcal{S}_1$ and $\mathcal{S}_2$.

(a) $\mathcal{S}_1$ has equation $2x+3y+4z = 10$, and $\mathcal{S}_2$ has equation $2x+3y-4z = 15$.

(b) $\mathcal{S}_1$ has equation $2x+3y+4z = 10$, and $\mathcal{S}_2$ has equation $-2x-3y-4z = 15$.