Homework 11: One More Problem – Solutions

1 For each of the following surfaces:

(i) Decide whether the surface is the graph of a function $f(x,y)$. If so, give a possible $f(x,y)$. If not, explain why not.

(ii) Decide whether the surface is a level set of a function $F(x,y,z)$. If so, give a possible $F(x,y,z)$. If not, explain why not.

(iii) Decide whether the surface has a parametric representation. If so, give a possible $\mathbf{r}(u,v)$. If not, explain why not.

Here are the surfaces:

(a) $z = x^2 + y^2$

Solution:

(i) This is a graph of $z = f(x,y) = x^2 + y^2$.

(ii) This is a level set: $F(x,y,z) = z - x^2 - y^2 = 0$.

(iii) This is a surface with a simple parametric representation: $\mathbf{r}(x,y) = \langle x, y, x^2 + y^2 \rangle$.

(b) $x = y^2 + z^2$

Solution:

(i) This is *not* the graph of a function.

(ii) This is a level set: $F(x,y,z) = x - y^2 - z^2 = 0$.

(iii) This is a surface with a simple parametric representation: $\mathbf{r}(y,z) = \langle y^2 + z^2, y, z \rangle$.

(c) $2x + 3y + 4z = 12$

Solution:

(i) This is a graph of $z = f(x,y) = \frac{1}{4}(12 - 2x - 3y)$.

(ii) This is a level set: $F(x,y,z) = 2x + 3y + 4z - 12 = 0$.

(iii) This is a surface with a simple parametric representation: $\mathbf{r}(x,y) = \langle x, y, \frac{1}{4}(12 - 2x - 3y)\rangle$. There are, of course, many others. This one can be re-written as

$$\mathbf{r}(x,y) = \langle 3, 0, 0 \rangle + x\langle 1, 0, -\frac{1}{2}\rangle + y\langle 0, 1, -\frac{3}{4}\rangle$$

which in turn can be written more simply as

$$\mathbf{r}(u,v) = \langle 3, 0, 0 \rangle + u\langle 2, 0, -1\rangle + v\langle 0, 4, -3\rangle.$$