1 Find the maximum and minimum of $f(x, y, z) = xyz^2$ on the filled ellipsoid $x^2 + 4y^2 + z^2 \leq 16$ in the following way:

(a) Find the values of $f(x, y, z)$ at the critical points inside the ellipsoid (so for points (x, y, z) with $x^2 + 4y^2 + z^2 < 16$).

Hint: You don't need to describe the set of all critical points, just find the value of $f(x, y, z)$ at these points.

(b) Use the technique of Lagrange multipliers to find the maximum and minimum value of $f(x, y, z)$ on the ellipsoid $x^2 + 4y^2 + z^2 = 16$.

(c) Combine your answers to parts (a) and (b) to find the maximum and minimum of $f(x, y, z)$ on the filled ellipsoid $x^2 + 4y^2 + z^2 \leq 16$.