Math 21a — **Final Exam** — Spring, 2009

Your Name

Your Signature

Instructions:

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☐	TTh 10	Jun Yin

- Please begin by printing and signing your name in the boxes above and by checking your section in the box to the right.
- You are allowed 3 hours (180 minutes) for this exam. Please pace yourself accordingly.
- You may not use any calculator, notes, or other assistance on this exam.
- In order to receive full credit, you must show your work and carefully justify your answers. The correct answer without any work will receive little or no credit.
- If you need more room, use the backs of the pages and indicate to the grader that you have done so.
- Please write neatly. Illegible answers will be assumed to be incorrect.
- Unless otherwise specified, you may use any valid method to solve a problem.
- Raise your hand if you have a question.
- Good luck!

Problem	Total Points	Score
1	5	
2	4	
3	6	
4	7	
5	8	
6	8	

Problem	Total Points	Score
7	14	
8	6	
9	11	
10	10	
11	10	
12	11	
Total	100	

1 (5 points) Indicate whether the following statements are True or False by circling the appropriate letter. No justifications are required.

T F The (vector) projection of $\langle 3, 17, -19\rangle$ onto $\langle 1, 2, 3\rangle$ is equal to the (vector) projection of $\langle 3, 17, -19\rangle$ onto $\langle -5, -10, -15\rangle$.

T F The angle between the vectors $\langle 1, -3, 7\rangle$ and $\langle -4, 6, 1\rangle$ is obtuse (greater than $\frac{\pi}{2}$).

T F If $\mathbf{F} = \langle P, Q\rangle$ is a vector field with the property that $Q_x - P_y = 0$, then Green's Theorem implies that $\int_C \mathbf{F} \cdot d\mathbf{r} = 0$ for any curve C.

T F The tangent plane to $x^2 - y^2 + 4z^2 = 1$ at the point $(1, 2, 1)$ is $x - 2y + 4z = 1$.

T F The two curves C_1, parameterized by $\mathbf{r}_1(t) = \langle t^2, t\rangle$, and C_2, parameterized by $\mathbf{r}_2(t) = \langle 4t^2, t\rangle$, both pass through the point $(0, 0)$. At this point, the curvature of C_1 is greater than the curvature of C_2.

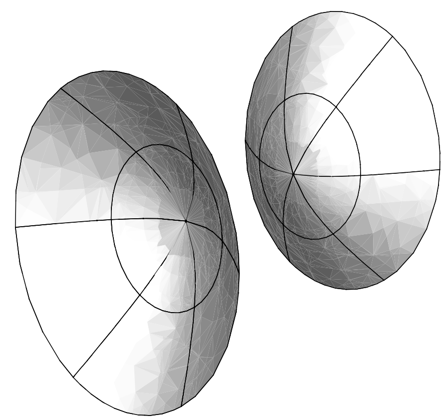
2 (4 points) Match the sketch of each quadric surface with the appropriate equation. You do not need to justify your choices.

(a) $x^2 + 2y^2 + z^2 = 1$

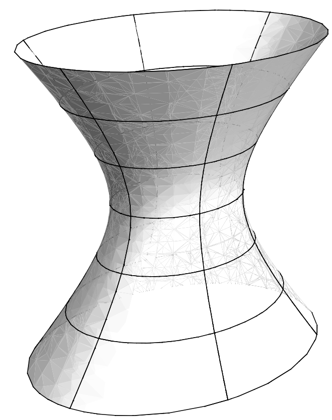
(b) $x^2 + 2y^2 - z^2 = 1$

(c) $x^2 - 2y^2 - z^2 = 1$

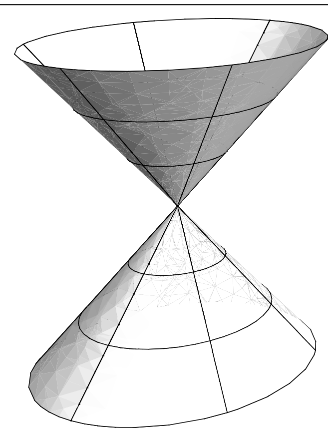
(d) $x^2 + 2y^2 - z^2 = 0$



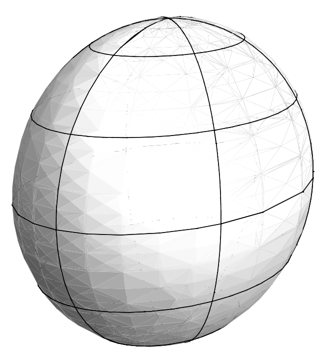
I. __________



II. __________



III. __________



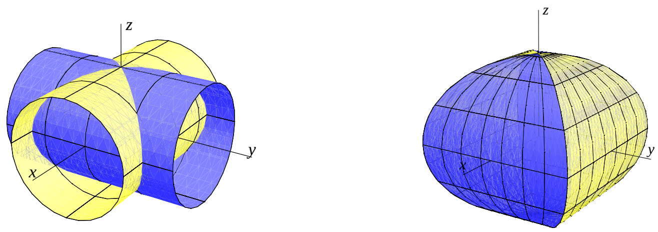
IV. __________

3 (6 points) Compute the integral

$$\int_1^2 \int_{2-x}^{\sqrt{2-x}} \frac{1}{2y^3 - 3y^2 + 4} \, dy \, dx.$$

Hint: Change the order of integration.

4 (7 points) Find the volume of the solid enclosed by the cylinders $x^2 + z^2 = 1$ and $y^2 + z^2 = 1$.



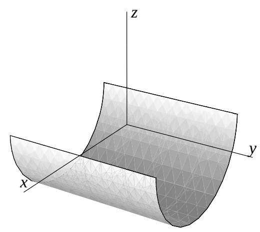
5 (8 points) Suppose the gradient vector of a function $f(x, y, z)$ at the point $(3, 4, -5)$ is $\langle 1, -2, 2\rangle$.

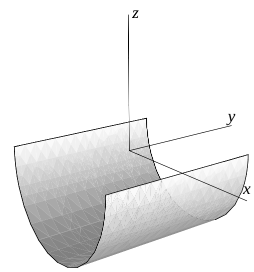
(a) (2 points) Find the values of the partial derivatives f_x, f_y, and f_z at the point $(3, 4, -5)$.

(b) (3 points) Find the maximum directional derivative of f at the point $(3, 4, -5)$ and the unit vector in the direction in which this maximum occurs.

(c) (3 points) If $f(3, 4, -5) = -2$, estimate $f(3.1, 4.1, -4.8)$ using linear approximation.

6 (8 points) Let S be the part of the elliptic cylinder $4x^2 + 9z^2 = 36$ lying between the planes $y = -3$ and $y = 3$ and below the plane $z = 0$. Here are two views of this cylinder, from different perspectives:





(a) (4 points) Write an iterated integral which gives the surface area of S. You need not evaluate the integral, but your integral should be simplified enough so that all that is required is integration (that is, the integrand contains no dot or cross products or even vectors).

(b) (4 points) Let $\mathbf{F}(x, y, z) = \langle x, e^y, z \rangle$. Evaluate the flux integral $\iint_S \mathbf{F} \cdot d\mathbf{S}$ if S is oriented with normal vectors pointing upward (that is, so the normal vectors have positive z component).

7 (14 points) Let C be the curve of intersection of the cylinder $x^2+y^2=4$ and the plane $3x+2y+z=6$.

(a) (2 points) Parameterize C. Be sure to give bounds for your parameter.

(b) (3 points) Write an integral that gives the arc length of C. You need not evaluate your integral, but your integral should be simplified enough so that all that is required is integration (that is, the integrand contains no dot or cross products or even vectors).

(c) (4 points) A bee is flying along the curve C in a room where the temperature is given by $f(x,y,z)=5x+5y+2z$. What is the hottest point the bee will reach?

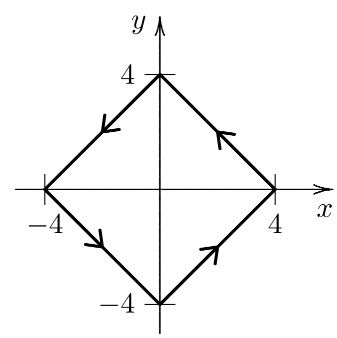
7 (Continued from previous page.) Recall that C is the curve of intersection of the cylinder $x^2+y^2=4$ and the plane $3x+2y+z=6$.

(d) (5 points) Let $\mathbf{F}(x,y,z)=\left\langle x^2y, e^{y^3}-xy^2, \cos z^2\right\rangle$. Evaluate the line integral $\int_C \mathbf{F}\cdot d\mathbf{r}$ if C is oriented counterclockwise when viewed from above.

8 (6 points) Evaluate the line integral

$$\int_C (x^4+3y)\,dx+(5x-y^3)\,dy,$$

where C is the boundary of the square with vertices $(4,0), (0,4), (-4,0)$, and $(0,-4)$, traversed counterclockwise.

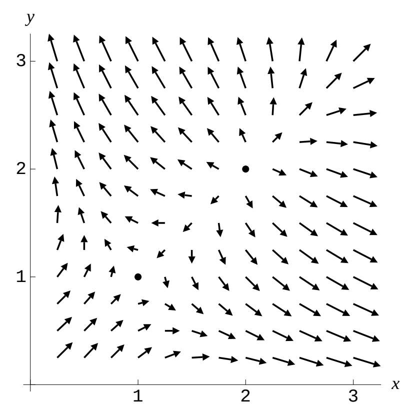


9 (11 points)

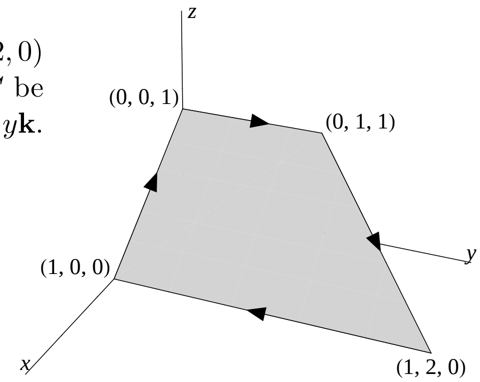
(a) (5 points) Find all critical points of $f(x, y) = x^2 + y^2 + x^2y + 4$, and classify each critical point as a local minimum, local maximum, or saddle point.

(b) (3 points) Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C be the straight line path from $(1, 1)$ to $(2, 2)$ and $\mathbf{F}(x, y)$ is the gradient vector field of $f(x, y)$ (from part (a)).

(c) (3 points) Here is the gradient vector field $\mathbf{G}(x, y)$ of another function $g(x, y)$. (Dots represent zero vectors.) Find all critical points of $g(x, y)$ in the region shown, and classify each critical point as a local minimum, local maximum, or saddle point.



10 (10 points) The four points $(1,0,0)$, $(0,0,1)$, $(0,1,1)$, and $(1,2,0)$ are all co-planar and form the vertices of a quadrilateral S. Let C be the boundary of S directed as shown to the right, and let $\mathbf{F} = y\mathbf{k}$.

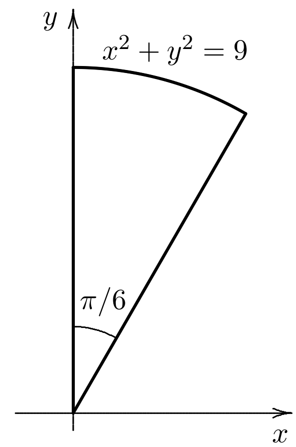


(a) (4 points) Find the equation of the plane that contains the quadrilateral S.

(b) (3 points) Calculate $\oint_C \mathbf{F} \cdot d\mathbf{r}$.

(c) (3 points) Calculate $\iint_S \operatorname{curl} \mathbf{F} \cdot d\mathbf{S}$.

11 (10 points)



(a) (4 points) Write $\iint_D f(x,y)\ dA$, where D is the region shown to the right, as an iterated integral of the form $\int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y)\, dy\, dx$.

(b) (4 points) Write $\iint_D f(x,y)\, dA$ as an iterated integral using polar coordinates.

(c) (2 points) Compute $\iint_D \cos(x^2+y^2)\, dA$.

12 (11 points) Let W be the solid tetrahedron with vertices $(0,0,0)$, $(1,0,0)$, $(0,1,0)$, and $(0,0,1)$. This tetrahedron has volume $\frac{1}{6}$.

(a) (4 points) Let S be the surface that forms the boundary of W, oriented with the outward-pointing normal. Note that S has four pieces. Compute the flux of the vector field $\mathbf{F}(x,y,z) = (x - 3yz)\mathbf{i} + (x^3 + y)\mathbf{j} + (\tan(xy) + z)\mathbf{k}$ out of the surface S. That is, compute $\iint_S \mathbf{F} \cdot d\mathbf{S}$.

(b) (3 points) Compute the flux of $\operatorname{curl} \mathbf{F}$ out of the surface S; that is, compute $\iint_S \operatorname{curl} \mathbf{F} \cdot d\mathbf{S}$.

(c) (4 points) Let S_1 be the part of S that lies in the yz-plane (where $x = 0$), oriented in the same way as S. Find the flux of $\mathbf{F}$ through S_1; that is, compute $\iint_{S_1} \mathbf{F} \cdot d\mathbf{S}$.