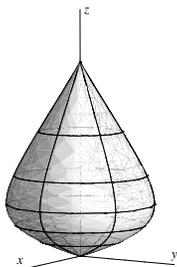


Practice Problems for Sections 13.6 - 13.9

1. Use the definition of the flux integral to evaluate $\iint_{\mathcal{S}} \vec{F} \cdot d\vec{S}$, where $\vec{F}(x, y, z) = \langle 9y, -x, z \rangle$ and \mathcal{S} is the half of the ellipsoid $x^2 + 9y^2 + 4z^2 = 16$ with $y > 0$, oriented with normals pointing in the positive y -direction.
2. Let C be the edges of the parallelogram with vertices $(0, 0, 0)$, $(1, 2, 3)$, $(2, 5, 3)$, and $(1, 3, 0)$, traveled counterclockwise when viewed from above. Let $\vec{F}(x, y, z) = \langle y, z^2, x \rangle$. Evaluate the line integral $\int_C \vec{F} \cdot d\vec{r}$.
3. True or false: If C is any curve in \mathbb{R}^3 and \vec{F} is a vector field on \mathbb{R}^3 whose curl is $\vec{0}$, then Stokes' Theorem implies that $\int_C \vec{F} \cdot d\vec{r} = 0$.
4. Let \mathcal{S} be the part of the plane $y = z$ contained inside the ellipsoid $4x^2 + y^2 + z^2 = 8$, oriented with normals pointing upward. Evaluate $\iint_{\mathcal{S}} \text{curl } \vec{F} \cdot d\vec{S}$, where $\vec{F}(x, y, z) = \langle 2x, z, 2x^2 + y^2 \rangle$.
5. Evaluate $\iint_{\mathcal{S}} \text{curl } \vec{F} \cdot d\vec{S}$, where $\vec{F}(x, y, z) = \langle y^2 + \sin yz, xz^3, x(y^2 + 1)^4 \rangle$ and \mathcal{S} is the ellipsoid $x^2 + 4y^2 + z^2 = 16$.
6. Evaluate $\iiint_E \text{div } \vec{F} \, dV$, where $\vec{F}(x, y, z) = \langle x^2y, -xy^2, -z \rangle$ and E is the solid ball $x^2 + y^2 + z^2 \leq 1$.
7. Find the volume of the solid enclosed by the parameterized surface $\vec{r}(u, v) = \langle (u - u^2) \cos v, (u - u^2) \sin v, u + u^4 \rangle$, $0 \leq u \leq 1$, $0 \leq v \leq 2\pi$.



8. Let \mathcal{S} be the part of the sphere $x^2 + y^2 + (z - 1)^2 = 2$ above the plane $z = 0$, oriented so that the unit normal vector at $(0, 0, 1 + \sqrt{2})$ is $\langle 0, 0, 1 \rangle$. Let $\vec{F}(x, y, z) = \langle 2xyz \cos z^2, -xy \sin xz, -\cos xz - y \sin z^2 \rangle$. Evaluate $\iint_{\mathcal{S}} \vec{F} \cdot d\vec{S}$.