1 Let S be the piece of the paraboloid $z = 1 - x^2 - y^2$ where $z \geq 0$. Let $\mathbf{n}$ denote the normal of S which points in the $+z$ direction at $(0, 0, 1)$. Let $\mathbf{F}$ denote the vector field

$$\mathbf{F} = \langle x + y\sin(z^2), y + x\sin(z^2), 1 - 2z \rangle.$$

(a) By parameterizing S, write down a double integral that computes the flux of $\mathbf{F}$ through S in the direction $\mathbf{n}$.

(b) Find a surface $\tilde{S}$ which is not S such that the fluxes of $\mathbf{F}$ through S and $\tilde{S}$ have the same absolute value.

(c) What is the value of the integral that you wrote down for part (a)?

2 Let $\mathbf{F} = \langle x + xz, y - yz, z^2 \rangle$. Here are two surfaces in $\mathbf{R}^3$ with the same boundary:

$$A: \quad z = \sqrt{1 - x^2 - y^2}, \qquad \text{with } x^2 + y^2 \leq 1$$
$$B: \quad z = 2\sqrt{1 - x^2 - y^2}, \qquad \text{with } x^2 + y^2 \leq 1.$$

(a) Which of the surfaces has the greatest flux of $\mathbf{F}$ through it? For both surfaces, use the normal which has a positive dot product with $\mathbf{k} = \langle 0, 0, 1 \rangle$.

(b) Which of the surfaces has the greatest flux of $\operatorname{curl}\mathbf{F}$ through it? For both surfaces, use the same normal as in part (a).

(c) Compute the flux of $\operatorname{curl}\mathbf{F}$ through each surface.

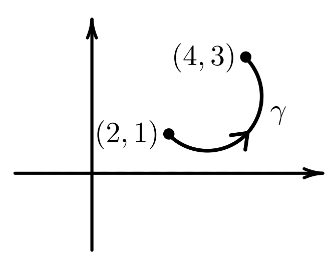
3 Find the volume of the solid bounded below by $z = 0$, above by $z^2 = 3x^2 + 3y^2$, and on the side by $x^2 + y^2 + z^2 = 4$.

4 Water is flowing down a vertical cylindrical pipe of radius 2 inches. The velocity vector field of the water at the outlet of the pipe is given by $\mathbf{v} = (r^2 - 4)\mathbf{k}$, where r is the distance in inches from the center of the pipe. How much water flows out of the bottom of the pipe in 3 seconds?

5 Calculate the work integral

$$\int_\gamma (x + y)\, dx + (3x - 2y)\, dy$$

for the curve γ shown, a semicircle from the point $(2, 1)$ to the point $(4, 3)$.



6 Compute the flux of the vector field $\mathbf{F}(x, y, z) = \left\langle e^{y^2+z^2}, y^2 + z^2, e^{x^2+y^2} \right\rangle$ across a portion of the cone with equation $4(x^2 + y^2) = 9z^2$ lying between $z = 0$ and $z = 2$ oriented with a downward normal.

7 Write down, but do not evaluate, a double integral that computes the surface area of the part of the surface $x^4 + 2x^2y^2 + y^4 + z^4 = 16$ with $x \geq 0$.

8 Let $\mathbf{F}$ be the vector field given by $\mathbf{F}(x,y,z) = \langle xy^2, xy^2, xy^2 \rangle$. Let D be the portion of the solid ball $x^2 + y^2 + z^2 \leq 9$ which lies in the first octant (that is, $x \geq 0$, $y \geq 0$, and $z \geq 0$). Set up, but do not evaluate, a triple integral in spherical coordinates which gives the flux of $\mathbf{F}$ out of the boundary of the region D.

9 Let $f(x,y,z)$ be a potential function for a conservative vector field $\mathbf{F}$; i.e., $\mathbf{F} = \nabla f$. Consider a level surface M for the function f, where M is given by $f(x,y,z) = k$, for some constant k. If C is a curve (not necessarily closed) on M, explain why $\displaystyle\int_C \mathbf{F} \cdot d\mathbf{r} = 0$.

10 Calculate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F} = \langle -e^y \sin x, e^y \cos x \rangle$ and C is the curve given by $\mathbf{r} = \langle \pi - \pi\cos t, \pi \sin t \rangle$ with $0 \leq t \leq \pi$.

11 Use Green's Theorem and the vector field $\mathbf{F} = \langle 0, x^3 y \rangle$ to compute the integral $\displaystyle\iint_R 3x^2 y \; dA$, where R is the region inside the ellipse $x^2 + \frac{y^2}{4} = 1$. For the boundary of the ellipse note that $\cos^2 t + \frac{(2\sin t)^2}{4} = 1$.

12 Let S be the surface given by $z = x^2 - y^2$. Let C be the curve on the surface S given by $x^2 + y^2 = 1$ and oriented counterclockwise as one looks down the z- axis. Use Stokes' Theorem to calculate the line integral $\displaystyle\oint_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F} = \langle x^2 + z^2, y, z \rangle$.

13 In appropriate units, the charge density $\sigma(x,y,z)$ in a region in space is given by $\sigma = \nabla \cdot \mathbf{E} = \operatorname{div} \mathbf{E}$, where $\mathbf{E}$ is the electrical field. Consider the unit cube given by $0 \leq x \leq 1$, $0 \leq y \leq 1$, and $0 \leq z \leq 1$. What is the total charge in this cube if

$$\mathbf{E} = \langle x(1-x)\ln(1+xyz), y(1-y)\tan(xyz), z(1-z)e^{xyz} \rangle.$$

You are being asked to integrate the charge density σ over the cube. You do not need to know any physics to do this problem.