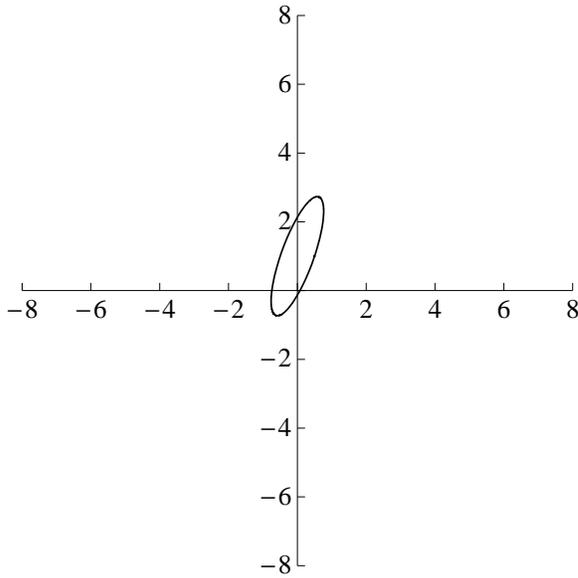


Problems for Review Session 2

- Parameterize the following curves. Be sure to give bounds for your parameter if appropriate.
 - The intersection of $x^2 + (z - 1)^2 = 1$ and $y = 4 - x^2 - z^2$.
 - The intersection of $z = e^{x+y}$ with $x = \sin y$.
 - The intersection of $4x^2 + 9y^2 + z^2 = 36$ with $z = 2y$.
 - The boundary of the surface \mathcal{S} , where \mathcal{S} is the portion of the paraboloid $z = x^2 + 4y^2$ underneath the plane $z = 5$.
- Parameterize the following surfaces. Describe the domain of your parameters (the possible values the parameters can take).
 - The portion of the sphere $x^2 + y^2 + z^2 = 1$ between the planes $z = -\frac{1}{2}$ and $z = 0$.
 - The part of the hyperboloid $9x^2 - y^2 + 4z^2 = -16$ with $y \geq 0$.
 - The ellipsoid $x^2 + 4y^2 + 9z^2 = 16$.
 - The part of the surface $y = x^2 - z^4$ lying inside the cylinder $x^2 + z^2 = 4$.
 - The part of the cylinder $y^2 + 4z^2 = 4$ lying between the planes $x + y = 2$ and $x = -2$.
- Find the point on the plane $x + 2y + 3z = -1$ closest to the point $(1, -3, -8)$.
- Let C be the level curve of $f(x, y) = \ln(x + 1) - y^2 + y^4$ passing through the point $(0, 1)$.
 - Find the arc length of the portion of C lying between the lines $y = -2$ and $y = 2$. (You may leave your answer as an integral.)
 - Let L be the line tangent to C at $(0, 1)$. Find the point at which L intersects the x -axis.
- Suppose $g(x, y, z)$ is a function for which $g(2, 1, 3) = 6$, $g_x(2, 1, 3) = 5$, $g_y(2, 1, 3) = -2$, and $g_z(2, 1, 3) = 4$.
 - Let S be the level surface of g that passes through the point $(2, 1, 3)$. Find the equation of the plane tangent to S at $(2, 1, 3)$.
 - Use linear approximation to approximate $g(2.1, 1.1, 3.1)$.
 - Suppose the surface in part (a) is the graph of a function $f(x, y)$. Evaluate $D_{\vec{u}}f(2, 1)$, where $\vec{u} = \langle \frac{3}{5}, \frac{4}{5} \rangle$.
 - Use linear approximation to approximate $f(2.1, 1.1)$.

6. Let C be the curve in \mathbb{R}^2 described by the equation $36x^2 - 24xy + 24x + 7y^2 - 14y = 2$, shown below:



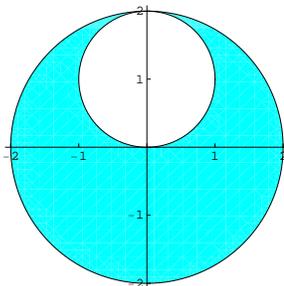
Let $f(x, y)$ be the distance from (x, y) to the curve C .

- In the picture above, sketch the level sets $f(x, y) = 0$, $f(x, y) = 1$ and $f(x, y) = 2$. Clearly indicate which is which.
- In the picture above, sketch the level set of f that passes through the point $(-6, 1)$ and the level set of f that passes through the point $(3, -3)$.
- At the point $(-6, 1)$, sketch the gradient vector $\nabla f(-6, 1)$. At the point $(3, -3)$, sketch the gradient vector $\nabla f(3, -3)$. What is the length of each of these gradient vectors?

Hint: The equation of the curve is not particularly important.

7. Find the maximum and minimum values of $f(x, y, z) = x + 2y + z$ on the surface $z = 4 - x^2 - y^2$, $z \geq 0$. (*Hint:* Parameterize the surface.)

8. (a) Evaluate $\iint_{\mathcal{R}} y \, dA$ where \mathcal{R} is the shaded region shown.



- (b) Evaluate $\int_1^5 \int_{\sqrt{y-1}}^2 e^{x^3} \, dx \, dy$.