Extra Problems on Double and Triple Integrals

1. A flat plate is in the shape of the region $\mathcal{R}$ described by $x^2 + y^2 \leq 4$. Its density at the point (x, y) is $f(x, y) = x^2 - 2x + 2y^2 + 2$, measured in grams per cubic cm.

   (a) Which point of the plate has the highest density? What is the density at that point?

   (b) Which point of the plate has the lowest density? What is the density at that point?

   (c) Find the mass of the plate.

   (d) Is your answer to (c) consistent with your answers to (a) and (b)? Explain.

2. A plot of land is shaped like the triangular region $\mathcal{R}$ with vertices $(0, 0)$, $(2, 0)$, and $(2, 3)$.

   (a) Vegetation density at the point (x, y) of the plot is given by the function $f(x, y) = y$. Use a double integral to find the amount of vegetation in the plot.

   (b) If you took Math 1b last semester, then you should be able to use the method of slicing to write a single integral that computes the amount of vegetation in the plot. How does that single integral relate to the double integral you found in (a)?

   (c) Suppose the vegetation density at the point (x, y) is instead given by the function $f(x, y) = ye^{x^3}$. Compute the amount of vegetation in the plot.

   (d) If you took Math 1b, can you do (c) using just what you learned in 1b? Why or why not?

3. Find the surface area of the portion of the plane $4x - y - 4z = 1$ which lies inside the paraboloid $y = x^2 + 4z^2$.

4. Suppose we'd like to know the surface area of the piece of the sphere $\rho = 10$ (in spherical coordinates) with $\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2} + 0.2$ and $\frac{\pi}{6} \leq \phi \leq \frac{\pi}{6} + 0.1$. Which of the following is the best estimate? (Hint: You don't need to do any integrating here; instead, think about how we came up with the surface area formula.)

   0.1   1   5   50

5. Let $\mathcal{U}$ be the solid enclosed by the surfaces $z = 0$, $z = x^2 y + 1$, $y = x^2$, and $y = 2 - x$. In this problem, you'll find the volume of $\mathcal{U}$ in two different ways.

   (a) Write a double integral that gives the volume of $\mathcal{U}$. Convert this to an iterated integral in the order $dy\, dx$.

   (b) Write a triple integral that gives the volume of $\mathcal{U}$. Convert this to an iterated integral in the order $dz\, dy\, dx$. How does this integral relate to the one you wrote down in (a)?

6. Let $\mathcal{U}$ be the solid bounded by $y = x^2 + 4z^2$ and $y = 4$. Write an iterated integral which gives the volume of $\mathcal{U}$.[1]

[1] You need not evaluate; it's a bit of a pain, no matter what order you use. However, if you know the formula for the area of an ellipse, you should be able to show that the volume is 4π.

7. Find the volume of the solid that lies within the sphere $x^2 + y^2 + z^2 = 4$, above the xy-plane, and below the cone $z = \sqrt{x^2 + y^2}$.

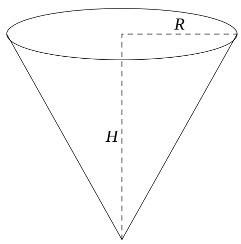
8. Let $\mathcal{U}$ be the solid $\sqrt{x^2 + y^2} \le z \le 4 - \sqrt{x^2 + y^2}$. The density of $\mathcal{U}$ at the point (x, y, z) is given by the function $f(x, y, z) = z$.

   (a) Let $\mathcal{S}$ be the surface of $\mathcal{U}$. Find the surface area of $\mathcal{S}$.

   (b) Find the volume of $\mathcal{U}$.

   (c) Find the mass of $\mathcal{U}$.

9. In this problem, you'll look at a right circular cone of height H and base radius R.



   (a) Show that the surface area of the cone is $\pi R\sqrt{R^2 + H^2} + \pi R^2$.

   (b) Show that the volume of the cone is $\frac{1}{3}\pi R^2 H$.