

(c) Now consider the space curve formed by the intersection of the same plane $y = z$ and the unit sphere (the sphere of radius one centered at the origin). Explain why this new space curve is parameterized by the vector function $\mathbf{p}(t)$.

- 6 (a) Sketch the graph of the function $f(x, y) = (y - 1)^2 - 1$. Make your sketch as neat as possible, and be sure to label your axes carefully.
- (b) Now sketch the graph of the function $f(x, y) = \sqrt{x^2 + y^2 - 1}$. Again, make your sketch as neat as possible, and label your axes carefully.

7 A moth is flying around a light bulb hanging from the ceiling of a room. The light bulb is 10 feet above the floor and the moth stays exactly 10 feet above the floor as it flies around the light bulb. The moth flies in a circular path, at constant speed, always exactly 2 feet away from the center of the light bulb.

- (a) Write down a set of equations that describe the circle traced out by the moth in spherical coordinates. Consider the origin to be the point on the floor directly below the light bulb.
- (b) Write down the equations giving the moth's circular path in cylindrical coordinates, and then give a parameterization of the moth's path in cylindrical coordinates.
- (c) Suppose that the moth's flight path is given by a vector function $\mathbf{r}(t)$, with parameter t representing time. Given what you know about the moth's flight answer the following questions:
- (i) Is it possible to determine $\mathbf{r}'(t) \cdot \mathbf{k}$? If so, what is it?
 - (ii) Describe the curve parameterized by $\mathbf{r}'(t) \times \mathbf{k}$.

8 Match the following space curves to the appropriate vector functions. (Some of the graphs appear on the next page.)

(a) $\mathbf{r}(t) = \langle \cos t, \sin t, \ln t \rangle$

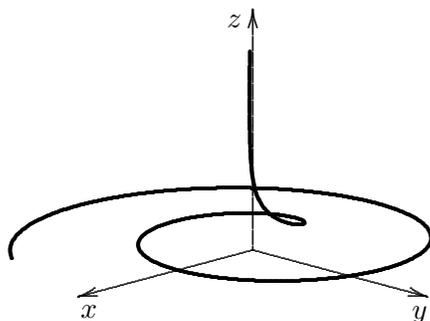
(b) $\mathbf{r}(t) = \langle t \cos t, t \sin t, \frac{1}{t} \rangle$

(c) $\mathbf{r}(t) = \langle \cos t, \sin t, \sin(3t) \rangle$

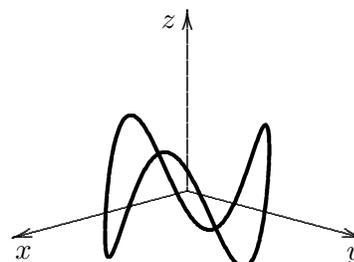
(d) $\mathbf{r}(t) = \langle t \cos t, t \sin t, t \rangle$

(e) $\mathbf{r}(t) = \langle \cos t, \sin t, t \rangle$

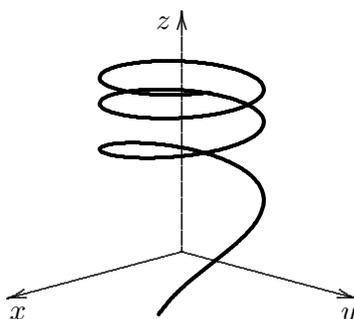
(f) $\mathbf{r}(t) = \langle \cos t, \sin t, \frac{1}{t} \rangle$



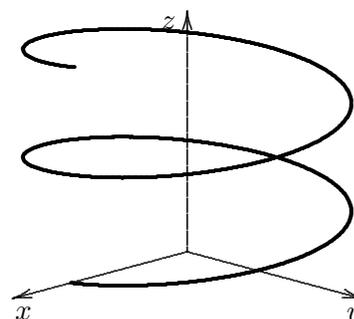
Curve I



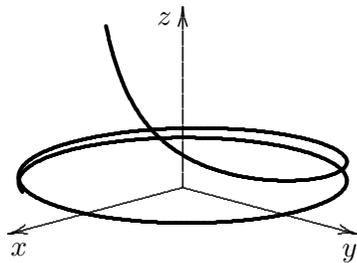
Curve II



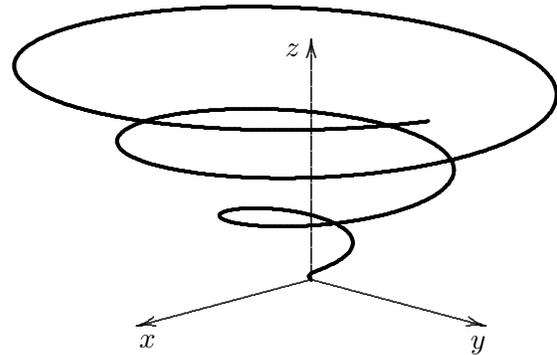
Curve III



Curve IV

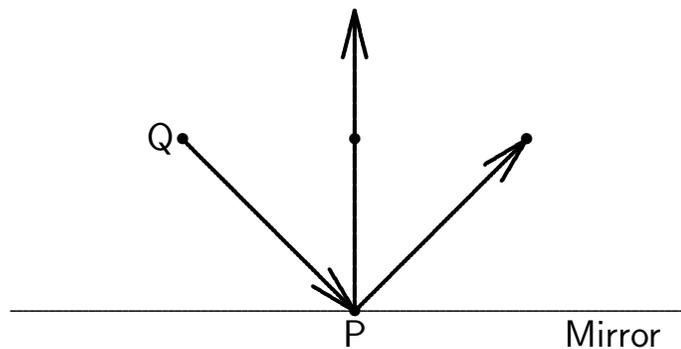


Curve V



Curve VI

- 9 A planar mirror in space contains the point $P(4, 1, 5)$ and is perpendicular to the vector $\mathbf{n} = \langle 1, 2, -3 \rangle$. The light ray $\overrightarrow{QP} = \mathbf{v} = \langle -3, 1, -2 \rangle$ with source $Q(7, 0, 7)$ hits the mirror plane at the point P .
- (a) Compute the projection $\mathbf{u} = \text{proj}_{\mathbf{n}}(\mathbf{v})$ of \mathbf{v} onto \mathbf{n} .
 - (b) Identify \mathbf{u} in the figure below and use it to find a vector parallel to the reflected ray.



- 10 What is the distance between the two cylinders $x^2 + y^2 = 1$ and $(z - 2)^2 + (x - 5)^2 = 4$?
- 11 (a) Parameterize the intersection of the ellipsoid

$$\frac{x^2}{4} + \frac{(y - 5)^2}{4} + \frac{z^2}{9} = 2$$

with the plane $z = 3$.

- (b) Parameterize the ellipsoid itself in the form $\mathbf{r}(\theta, \phi) = \dots$.
- (c) What is the curvature of the curve in part (a) at the point $(2, 0, 3)$.

Hint: While you may use the curvature formula $\kappa(t) = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}$, you are also allowed to cite a fact which you know about the curvature.