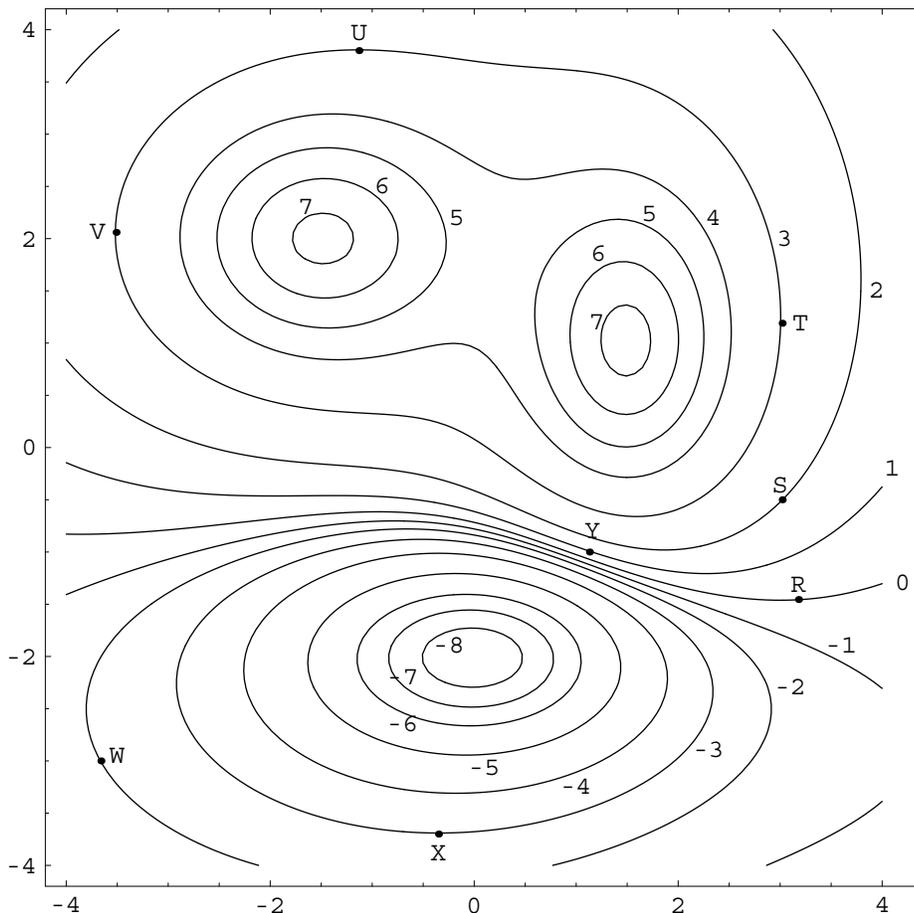


PART I: Multiple choice. Each problem has a unique correct answer. You do not need to justify your answers in this part of the exam.

- 1 The vectors $\mathbf{A} = -2\mathbf{i} + (t - 1)\mathbf{j} + 2\mathbf{k}$ and $\mathbf{B} = \mathbf{j} + t\mathbf{k}$ are parallel when:
- $t = 0$;
 - $t = \frac{1}{3}$;
 - $t = 1$ and $t = 2$;
 - all values of t ;
 - no values of t .
- 2 If \mathbf{A} and \mathbf{B} are vectors in space, then the expression $(\mathbf{A} + \mathbf{B}) \cdot (\mathbf{A} \times \mathbf{B})$ equals
- $\mathbf{0}$ (the zero vector) because $\mathbf{A} + \mathbf{B}$ and $\mathbf{A} \times \mathbf{B}$ are always parallel;
 - 0 (the number zero) because \mathbf{A} and \mathbf{B} are always perpendicular to $\mathbf{A} \times \mathbf{B}$;
 - a positive number since $(\mathbf{A} + \mathbf{B}) \cdot (\mathbf{A} \times \mathbf{B})$ is the volume of a parallelepiped;
 - undefined since we cannot take the dot product of a vector and a scalar;
 - an arbitrary number, depending on the particular vectors \mathbf{A} and \mathbf{B} .
- 3 The curve $\mathbf{r}(t) = \langle e^{\sqrt{2}t} \cos t, e^{\sqrt{2}t} \sin t \rangle$, $0 \leq t \leq \pi$, has length
- $\frac{1}{\sqrt{3}} (e^{\sqrt{2}\pi} - 1)$;
 - $\frac{1}{\sqrt{2}} (e^{\sqrt{2}\pi} - 1)$;
 - $\sqrt{\frac{2}{3}} (e^{\sqrt{2}\pi} - 1)$;
 - $\sqrt{\frac{3}{2}} (e^{\sqrt{2}\pi} - 1)$;
 - given by a definite integral which cannot be evaluated explicitly;
 - given by an indefinite integral.

The questions in problems 4-5 refer to the marked points in the diagram below, which shows the level curves of a function $f(x, y)$.

- 4 At which of the labeled points is the partial derivative $\frac{\partial f}{\partial x}$ strictly positive?
- | | |
|--------------------|--------------------|
| (a) at S and T | (b) at S and W |
| (c) at T and W | (d) at V and W |
| (e) at V and Y | (f) only at V |
| (g) at W and Y | (h) only at T |
- 5 At the point Y , what can one say about the values of $\frac{\partial f}{\partial y}$ and $\frac{\partial^2 f}{\partial y^2}$?
- | | |
|---|---|
| (a) $\frac{\partial f}{\partial y} > 0$ and $\frac{\partial^2 f}{\partial y^2} > 0$ | (b) $\frac{\partial f}{\partial y} > 0$ and $\frac{\partial^2 f}{\partial y^2} < 0$ |
| (c) $\frac{\partial f}{\partial y} < 0$ and $\frac{\partial^2 f}{\partial y^2} > 0$ | (d) $\frac{\partial f}{\partial y} < 0$ and $\frac{\partial^2 f}{\partial y^2} < 0$ |
| (e) $\frac{\partial f}{\partial y} = 0$ and $\frac{\partial^2 f}{\partial y^2} > 0$ | (f) $\frac{\partial f}{\partial y} = 0$ and $\frac{\partial^2 f}{\partial y^2} < 0$ |
| (g) $\frac{\partial f}{\partial y} > 0$ and $\frac{\partial^2 f}{\partial y^2} = 0$ | (h) $\frac{\partial f}{\partial y} < 0$ and $\frac{\partial^2 f}{\partial y^2} = 0$ |



6 A particle moves in space with position vector $\mathbf{r}(t)$. If $\mathbf{r}''(t) \perp \mathbf{r}'(t)$ for all t , what can you conclude about the motion?

- (a) $\mathbf{r}(t) \times \mathbf{r}'(t)$ is constant;
- (b) $\mathbf{r}'(t) \times \mathbf{r}''(t)$ is constant;
- (c) $|\mathbf{r}(t)|^2 = \mathbf{r}(t) \cdot \mathbf{r}(t)$ is constant;
- (d) $|\mathbf{r}'(t)|^2 = \mathbf{r}'(t) \cdot \mathbf{r}'(t)$ is constant;
- (e) $\mathbf{r}'(t)$ is constant;
- (f) $\mathbf{r}''(t)$ is constant;

PART II: Free response questions. You should attempt all parts of each problem. Show your work!

7 Let Π_1 be the plane described by the equation $x - y + z = 5$ and Π_2 the plane passing through the three points $(2, 1, 0)$, $(1, 1, 1)$, $(0, 0, 1)$.

- (a) Find an equation describing the plane Π_2 .
- (b) Show that the planes Π_1 and Π_2 are parallel.
- (c) Find the distance between Π_1 and Π_2 . (Hint: The distance between two parallel planes equals the distance between any particular point on one plane to the other plane.)

8 An airplane traces out the helix $x = \cos t$, $y = \sin t$, $z = 2\pi - t$, ($0 \leq t \leq 2\pi$). As it travels, it emits a laser ray shining straight ahead – that is, shining in the direction of the velocity vector.

- Find the velocity vector of the airplane at time t .
- Give a linear parameterization of the laser ray emitted at time $t = \pi$. Where does this ray intersect the xy -plane?
- Give a linear parameterization of the laser ray emitted at time t . (Now t is not a specific number!) Hint: use a different letter to denote the parameter for the tangent line.
- As the airplane travels along the helix, the spots where the laser rays intersect the xy -plane trace out a curve in the xy -plane. Give a parameterization of this plane curve.

9 (a) In *only a few words*, describe the shape of the surface S , given parametrically by the equation

$$\mathbf{r}(\theta, t) = \left\langle \frac{t}{2} \cos \theta, \frac{t}{3} \sin \theta, t^2 \right\rangle, \quad -\pi \leq \theta < \pi, \quad t \geq 0.$$

- Verify that the point $P = (\frac{1}{2}, \frac{1}{3}, 2)$ lies on the surface S .
- Give an alternate description of the surface S of part (a) as the graph of a function of x and y . That is, find an equation $z = f(x, y)$ in terms of the Euclidean coordinates x, y, z , but not involving the parameters θ, t . Hint: you may find the relation $\cos^2 \theta + \sin^2 \theta = 1$ useful.
- Find a linear equation for the tangent plane to S at the point P of part (b). You can do this using either the parameterization of S in part (a), or your answer to part (c).

10 Find an equation for the plane that passes through the origin and whose normal vector is parallel to the line of intersection of the planes $2x + y + z = 4$ and $x + 3y + z = 2$.

11 For each of the following, fill in the blank with $<$ (less than), $>$ (greater than), or $=$ (equals). Justify your answer completely.

(a) The arc length of the curve parameterized by $\mathbf{f}(t) = \langle \cos(2t), 0, \sin(2t) \rangle$, $0 \leq t \leq \pi$.

The arc length of the curve parameterized by $\mathbf{g}(u) = \langle 3, 2 \cos(u^2), 2 \sin(u^2) \rangle$, $0 \leq u \leq \sqrt{\pi}$.

(b) The arc length of the curve parameterized by $\mathbf{f}(t) = \langle t^2, 2 \cos(t), 2 \sin(t) \rangle$, $0 \leq t \leq 2\pi$.

The arc length of the curve parameterized by $\mathbf{g}(u) = \langle u^4, 2 \cos(u^2), 2 \sin(u^2) \rangle$, $0 \leq u \leq 2\pi$.

(c) The arc length of the curve parameterized by $\mathbf{f}(t) = \langle 1 + 3t^2, 2 - t^2, 5 + 2t^2 \rangle$, $0 \leq t \leq 1$.

The arc length of the curve parameterized by $\mathbf{g}(u) = \langle \frac{1}{2}u^2, u, \frac{2\sqrt{2}}{3}u^{3/2} \rangle$, $0 \leq u \leq 2$.

(d) The arc length of the curve parameterized by $\mathbf{f}(t) = \langle \sin t, \cos t, t \rangle$, $1 \leq t \leq 5$.

The arc length of the curve parameterized by $\mathbf{g}(u) = \langle u \sin u, u \cos u, u \rangle$, $1 \leq u \leq 5$.

12 Find the distance between the cylinder $x^2 + y^2 = 1$ and the line L given by the symmetric equations

$$\frac{x+2}{4} = \frac{y-1}{3} = \frac{z}{2}.$$