OLD EXAM ONE – SPRING 2008

PART I: Multiple choice. Each problem has a unique correct answer. You do not need to justify your answers in this part of the exam.

1 Which of the following statements are true?

(a) If $\mathbf{a} \cdot \mathbf{b} = \mathbf{a} \cdot \mathbf{c}$, then $\mathbf{b} = \mathbf{c}$.

(b) If $\mathbf{b} = \mathbf{c}$, then $\mathbf{a} \cdot \mathbf{b} = \mathbf{a} \cdot \mathbf{c}$.

(c) $\mathbf{a} \cdot \mathbf{a} = |\mathbf{a}|^2$

(d) If $|\mathbf{a}| > |\mathbf{b}|$, then $\mathbf{a} \cdot \mathbf{c} > \mathbf{b} \cdot \mathbf{c}$.

(e) If $|\mathbf{a}| = |\mathbf{b}|$, then either $\mathbf{a} = \mathbf{b}$ or $\mathbf{a} = -\mathbf{b}$.

(f) If $\mathbf{a} \times \mathbf{b} = \mathbf{a} \times \mathbf{c}$, then $\mathbf{b} = \mathbf{c}$.

(g) $\mathbf{a} \times \mathbf{a} = |\mathbf{a}|^2$

(h) $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{b}) \times \mathbf{c}$

2 Match each contour plot with one of the following functions.

(a) $f(x, y) = x^2 - y^2$

(b) $f(x, y) = xy^2$

(c) $f(x, y) = y^2 - x^2$

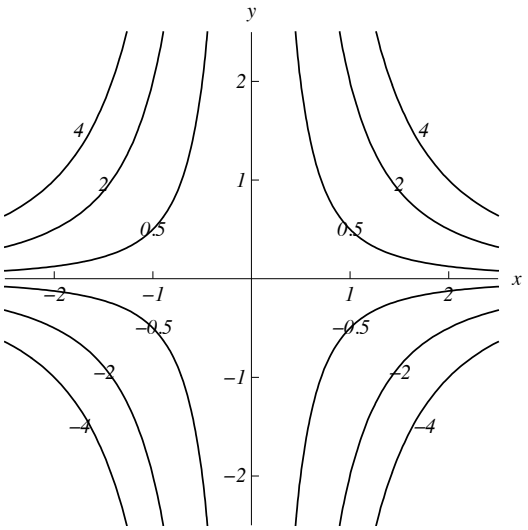
(d) $f(x, y) = x^2 y^2$

(e) $f(x, y) = x - y^2$

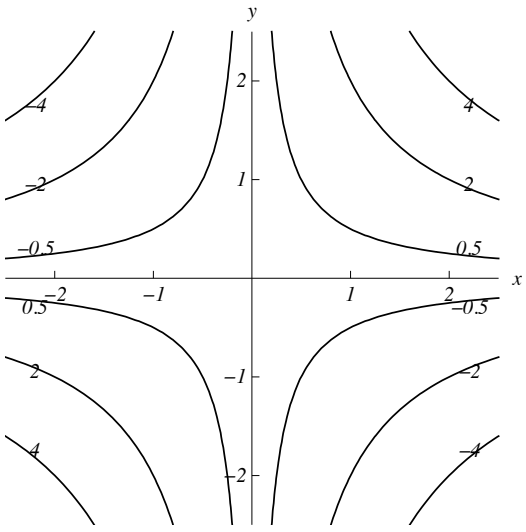
(f) $f(x, y) = xy$

(g) $f(x, y) = x^2 y$

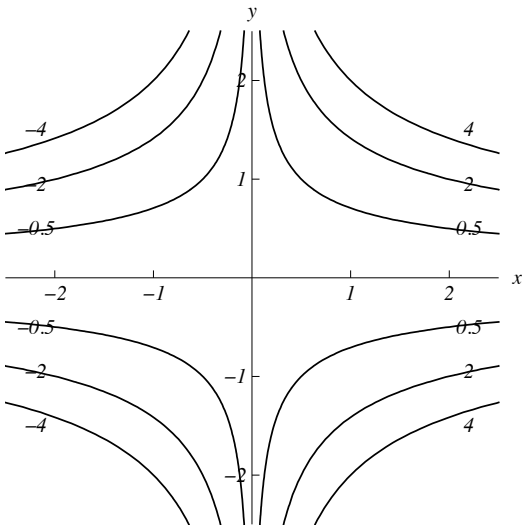
(h) $f(x, y) = y - x^2$



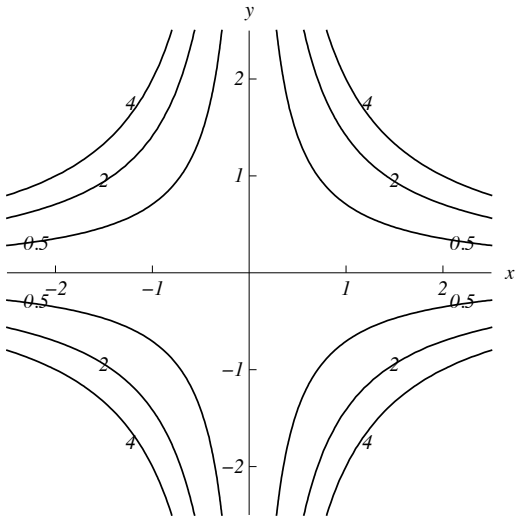
(i)



(ii)



(iii)



(iv)

3 Match each of the following equations with the appropriate graph below.

(a) $x^2 + 4y^2 + 9z^2 = 1$

(b) $x^2 - y^2 + z^2 = 1$

(c) $y = 2x^2 + z^2$

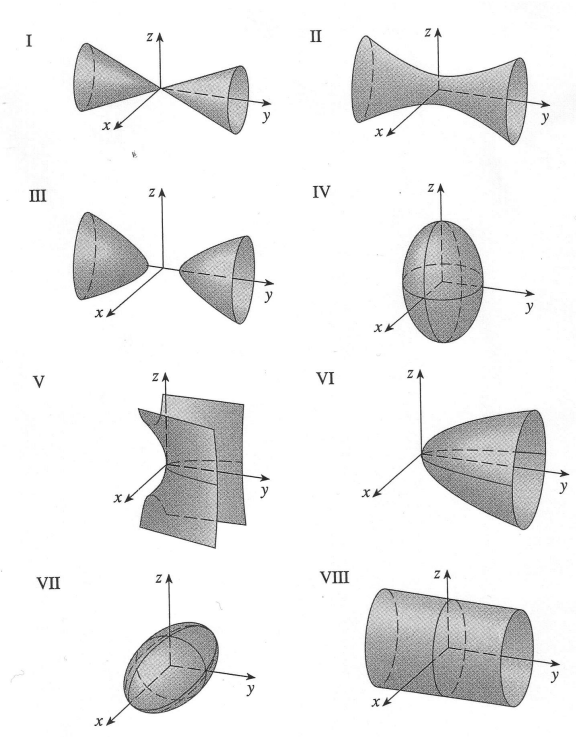
(d) $x^2 + 2z^2 = 1$

(e) $9x^2 + 4y^2 + z^2 = 1$

(f) $-x^2 + y^2 - z^2 = 1$

(g) $y^2 = x^2 + 2z^2$

(h) $y = x^2 - z^2$



4 The volume of the parallelepiped determined by the vectors $\mathbf{u} = 3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$, $\mathbf{v} = -2\mathbf{i} + 5\mathbf{j} + \mathbf{k}$, and $\mathbf{w} = 2\mathbf{i} + \mathbf{j} + 5\mathbf{k}$ is...

(a) 54 (b) 36 (c) 108

(d) 144 (e) None of the above.

5 Match each equation below with the corresponding equation in polar, spherical, or cylindrical coordinates.

(a) $x^2 + y^2 + z^2 = x$

(b) $(x^2 + y^2)^2 = x^2 - y^2$

(c) $x^2 + y^2 = z - 3$

(d) $x^2 + y^2 - z^2 = 1$

(e) $x^2 - 2xy + y^2 = z^2$

(i) $r^2(1 - \sin 2\theta) = z^2$

(ii) $r^2 = z - 3$

(iii) $r^2 = \cos 2\theta$

(iv) $\rho = \cos\theta \sin\phi$

(v) $\rho^2 \cos 2\phi = -1$

6 The function

$$f(x, y) = \frac{3x - 4y}{x^2 + y^2}.$$

satisfies which of the following partial differential equations.

(a) $f_{xx} = f_{yy}$

(b) $f_x = f_{yy}$

(c) $f_{xx} + f_{yy} = 0$

(d) $f_y = f_{xx}$

(e) None of the above.

7 Assume that $\mathbf{u}$ and $\mathbf{v}$ are *unit* vectors. For each of the following assumptions, determine whether the two vectors must be (i) perpendicular, (ii) pointing in the same direction, (iii) pointing in opposite directions, or (iv) parallel but one cannot tell from the given information whether they point in the same or opposite directions:

(a) $\mathbf{u} \cdot \mathbf{v} = 1$

(b) $\mathbf{u} \cdot \mathbf{v} = 0$

(c) $\mathbf{u} \cdot \mathbf{v} = -1$

(d) $|\mathbf{u} \cdot \mathbf{v}| = 1$

(e) $|\mathbf{u} \cdot \mathbf{v}| = 0$

(f) $|\mathbf{u} \times \mathbf{v}| = 1$

(g) $|\mathbf{u} \times \mathbf{v}| = 0$

PART II: Free response questions. You should attempt all parts of each problem. Show your work!

8 Let

$$A = (1, 0, 0) \qquad B = (1, 1, 2) \qquad C = (1, 2, 1) \qquad D = (2, 2, 3).$$

Find

(a) The distance from D to A.

(b) The distance from D to the line containing A and B.

(c) An equation of the plane containing A, B, and C.

9 An astronaut is flying in a spacecraft along the path described by

$$\mathbf{r}(t) = \langle t^2 - t, 2 + t, -3/t \rangle,$$

where t is given in hours.

(a) What is the velocity of the spacecraft when it reaches the point $(6, 5, -1)$?

(b) What is the speed of the spacecraft when it reaches the point $(6, 5, -1)$?

(c) What is the acceleration of the spacecraft when it reaches the point $(6, 5, -1)$?

(d) If the engines of the spacecraft are shut off when it reaches the point $(6, 5, -1)$, where will the spacecraft be 2 hours later?

10 Consider the parameterized curve $\mathbf{r}(t) = \langle 12t, 5\cos t, 3 - 5\sin t \rangle$. Find the arc length from $t = 0$ to $t = 2$.

11 Find $\mathbf{r}(t)$ if $\mathbf{r}'(t) = (2t + 1)\mathbf{i} + \cos t\mathbf{j} - e^t\mathbf{k}$ and $\mathbf{r}(0) = -\mathbf{i} + \pi\mathbf{j} + 3\mathbf{k}$.

12 (a) Find a parameterization of the line of intersection of the planes $3x - 2y + z = 7$ and $x + 2y + 3z = -3$.

(b) Find the symmetric equations

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$$

representing this line.

13 The intersection of the two surfaces $x^2 + \frac{y^2}{2} = 1$ and $z^2 + \frac{y^2}{2} = 1$ consists of two curves.

(a) Parameterize each curve in the form $\mathbf{r}(t) = \langle x(t), y(t), z(t) \rangle$.

(b) Set up the integral for the arc length of one of the curves.

(c) What is the arc length of this curve?

14 Imagine the planet Earth as the unit sphere centered at the origin in three-dimensional space. An asteroid is approaching from the point $P(0, 4, 3)$ along the path

$$\mathbf{r}(t) = \langle (4 - t)\sin(t), (4 - t)\cos(t), 3 - t \rangle.$$

(a) When and where will it first hit the earth?

(b) What velocity will it have at impact?