Exam One Review

1 Suppose that $\mathbf{a}$ and $\mathbf{b}$ are two vectors such that $\mathbf{b}$ is twice as long as $\mathbf{a}$. If the angle between $\mathbf{a}$ and $\mathbf{b}$ is 60 degrees, calculate the value of

$$\frac{(\mathbf{a}+2\mathbf{b})\cdot(\mathbf{a}-\mathbf{b})}{2|\mathbf{a}|^2}.$$

2 Let $\mathbf{r}_1(t) = \langle t, 2, 1\rangle$ and $\mathbf{r}_2(t) = \langle t, 1, -2\rangle$ be two vector-valued functions, where t represents time in seconds. Consider the the parallelogram determined by by the vectors $\mathbf{r}_1(t)$ and $\mathbf{r}_2(t)$. What is the area of this parallelogram when $t = 1$? What is the rate at which the area is changing at this time?

3 Find the equation of a plane that passes through points $P(5, 3, 2)$ and $Q(3, 1, 0)$ and that never intersects the y-axis.

4 Consider two planes with normal vectors $\mathbf{n}_1 = \langle 20, 40, 0\rangle$ and $\mathbf{n}_2 = \langle 0, 26, -26\rangle$, respectively. Suppose that both planes contain the point $(1, 1, 1)$. Find a vector equation that parameterizes the line of intersection of these two planes.

5 A pitcher on the Math 21a baseball team gets ready to pitch a baseball. During the pitch, the pitchers hand traces a curve given by the parametric equations

$$\mathbf{x}(t) = \langle \cos t, \sin t, 6 - 2t\rangle.$$

The pitcher begins the pitch at $t = 0$ and lets go of the ball when $t = \pi/2$. How far does the ball in the pitchers hand travel along this curve during this time? Where is the ball 2 seconds after it leave the pitchers hand?

6 Find a parameterization for the curve of the intersection of the cylinder $(x - 1)^2 + z^2 = 1$ and the plane $x + y + z = 1$.

7 Suppose that a somewhat strange roller coaster ride follows the path described by the vector function

$$\mathbf{r}(t) = \langle -2\sin t, -2\sin t, 2\cos t\rangle,$$

where t is measured in seconds. The ride lasts for π seconds. At what times during the ride is the roller coaster either going up at a 30 degree angle or going down at a 30 degree angle?

8 Which of the following regions in $\mathbf{R}^3$ are equivalent?

(a) $\rho \le 1$ and $0 \le \phi \le \frac{\pi}{4}$

(b) $z \le 1$ and $r^2 \ge z^2$

(c) $r^2 + z^2 \le 1$ and $r^2 \ge z^2$

(d) $x^2 + y^2 + z^2 \le 1$ and $x^2 + y^2 \le z^2$

(e) $\rho \le 1$ and $\frac{\pi}{4} \le \phi \le \frac{3\pi}{4}$

9 Consider the parameterized surface

$$\mathbf{r}(u, v) = \langle \cos v, \sin v, u\rangle.$$

Write equations describing this surface in rectangular, cylindrical, and spherical coordinates.

10 (a) Find the curvature $\kappa(t)$ of the space curve $\mathbf{r}(t) = \langle -\cos t, \sin t, -2t\rangle$ at the point $\mathbf{r}(0)$.

(b) Find the curvature $\kappa(t)$ of the space curve $\mathbf{r}(u) = \langle -\cos(5t), \sin(5t), -10t\rangle$ at the point $\mathbf{r}(0)$.

11 (a) Find the surface S that is the set of points $P(x, y, z)$ for which the distance from P to $A(1, 2, 3)$ is equal to the distance from P to $B(5, 8, 5)$.

(b) Find the distance from A to S.

12 Find the distance between the cylinder $x^2 + y^2 = 1$ and the line given in symmetric form by

$$\frac{x+2}{4} = \frac{y-1}{3} = \frac{z}{2}.$$

13 Find the arc length of the curve

$$\mathbf{r}(t) = \langle e^t + e^{-t}, 2\cos t, 2\sin t\rangle$$

from $t = 0$ to $t = 4$.

14 Some true-false questions:

(a) **T** **F** There is a vector $\mathbf{v}$ for which the vector projection $\text{proj}_{\mathbf{v}}\,\mathbf{j}$ is equal to $2\mathbf{j}$.

(b) **T** **F** The length of the unit tangent vector $\mathbf{T}$ for a curve $\mathbf{r}(t)$ is independent of t.

(c) **T** **F** If $\mathbf{v} \times \mathbf{w} = \mathbf{0}$, then $\mathbf{v} = \mathbf{0}$ or $\mathbf{w} = \mathbf{0}$.

(d) **T** **F** The set of points P for which the distance from P to the point $(0, 0, 0)$ is 1 less than the distance to the point $(0, 0, 2)$ is an elliptic paraboloid.

(e) **T** **F** The curvature of a curve depends on the speed at which one travels upon it.

(f) **T** **F** If the speed of a parameterized curve is constant, then the curvature of this curve is zero.

(g) **T** **F** The surface $z = \sin(x)$ contains lines which are parallel to the y-axis.

(h) **T** **F** For the moving frame $\{\mathbf{T}, \mathbf{N}, \mathbf{B}\}$, we must always have $\mathbf{B} \cdot (\mathbf{T} \times \mathbf{N}) = 1$.

(i) **T** **F** The function $f(x, y, z) = x^2 + y^2 + z^2$ satisfies the partial differential equation $f_x^2 + f_y^2 + f_z^2 = 6f$.

(j) **T** **F** There is a quadric surface which has both hyperbolas and parabolas as traces.

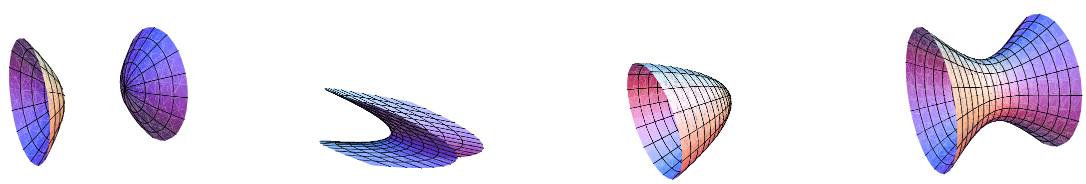
15 Match the equations to the graphs of the surfaces:

(a) $x + y^2 - z^2 - 1 = 0$

(b) $-x^2 + y^2 + z^2 - 1 = 0$

(c) $-x^2 + y^2 + z^2 + 1 = 0$

(d) $-x + y^2 + z^2 + 1 = 0$



Surface I   Surface II   Surface III   Surface IV

16 Match the parameterized curves to their equations:

(a) $\mathbf{r}(t) = \langle \cos t, \sin t\rangle$

(b) $\mathbf{r}(t) = \langle \cos t, t\rangle$

(c) $\mathbf{r}(t) = \langle \cos t, \cos^2 t\rangle$

(d) $\mathbf{r}(t) = \langle \cos t, \sin(2t)\rangle$