$\boxed{1}$ (8 points)

(a) (4 points) Find an equation for the plane containing the three points $P(3, 3, 1)$, $Q(2, -1, 0)$, and $R(-1, -3, 1)$.

Solution: One normal for this plane is $\overrightarrow{PQ} \times \overrightarrow{PR} = \langle -1, -4, -1\rangle \times \langle -4, -6, 0\rangle = \langle -6, 4, -10\rangle$. Thus various equations for the plane are $\boxed{\langle -6, 4, -10\rangle \cdot \langle x-3, y-3, z-1\rangle = 0}$ or $\boxed{-6x + 4y - 10z = -16}$ or $\boxed{3x - 2y + 5z = 8}$.

This type of problem (finding the plane containing three given points) appeared on your homework (§9.5, #26) as well as on the practice exams (#7(a) on the practice exam from Spring 2007 and #8(c) on the practice exam from Spring 2008).

(b) (4 points) Are the four points P, Q, R, and $S(7, 4, -1)$ coplanar? (Here P, Q, and R are the points from part (a).) Justify your answer.

Solution: In part (a) we found the plane determined by P, Q, and R. Now this question can be rephrased as: "Does the point S lie on this plane?" We can simply plug in and check: does $3(7) - 2(4) + 5(-1) = 8$? Yes, so $\boxed{\text{the points are coplanar}}$.

Of course, you could also use the scalar triple product to do this problem, as you did in the homework problem §9.4, #26.

$\boxed{2}$ (12 points)

(a) (4 points) Find an equation for the plane given by the parameterization

$$\mathbf{r}(u, v) = \langle 3 + 2u, 5 - u + v, 2u + 3v\rangle .$$

Solution: If we re-write this parameterization as

$$\mathbf{r}(u, v) = \langle 3, 5, 0\rangle + u\langle 2, -1, 2\rangle + v\langle 0, 1, 3\rangle,$$

we can see the point $(3, 5, 0)$ lies on the plane and the vectors $\langle 2, -1, 2\rangle$ and $\langle 0, 1, 3\rangle$ are parallel to it. Thus a normal vector $\mathbf{n}$ is

$$\mathbf{n} = \langle 2, -1, 2\rangle \times \langle 0, 1, 3\rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -1 & 2 \\ 0 & 1 & 3 \end{vmatrix} = \langle -5, -6, 2\rangle.$$

Thus equations for the plane include $\boxed{\langle -5, -6, 2\rangle \cdot \langle x-3, y-5, z\rangle = 0}$ and $\boxed{-5x - 6y + 2z = -45}$.

If you aren't familiar with this trick, another way to find the plane is to simply plug in several values of the pair (u, v) and find three points on the plane. From this we find two vectors and proceed as above.

(b) (3 points) Suppose the curve C is parameterized with respect to arc length by $\mathbf{r}(t)$ (that is, this parameterization has $|\mathbf{r}'(t)| = 1$ for all t). What is the distance along C between $\mathbf{r}(3)$ and $\mathbf{r}(10)$?

Solution: The distance along C is simply the arc length:

$$\int_3^{10} |\mathbf{r}'(t)|\, dt = \int_3^{10} 1\, dt = t\Big|_3^{10} = 10 - 3 = \boxed{7}.$$

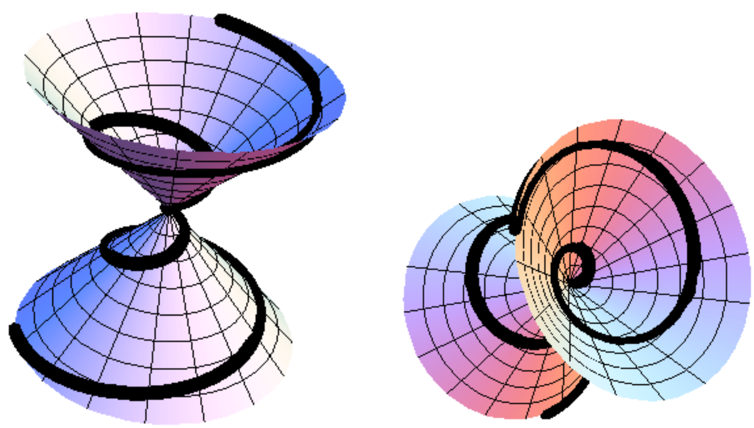
(c) (2 points) Suppose the traces of a quadric surface are parabolas ($x = k$), parabolas ($y = k$), and hyperbolas ($z = k$). What quadric surface is this? Explain your reasoning.

Solution: This is a $\boxed{\text{hyperbolic paraboloid}}$. It's a paraboloid because traces in two directions are parabolas, and since the trace in the third direction is a hyperbola, it's a hyperbolic paraboloid.

(d) (3 points) An ant is standing on the surface $z = x^3 - 3xy + e^{xy}$ at the point $(1, 0, 2)$. If the ant walks East (that is, in the positive x direction), is he moving up or down? Explain your reasoning.

Solution: If the ant is moving in the x direction, then his height z is changing at the rate $\frac{\partial z}{\partial x}$. We compute this partial derivative to see that $\frac{\partial z}{\partial x} = 3x^2 - 3y + ye^{xy}$. At the point $(x, y) = (1, 0)$, this is $3(1)^2 - 3(0) + 0e^{(1)(0)} = 3$. Since this is positive, $\boxed{\text{the ant is moving up}}$.

$\boxed{3}$ (12 points) Consider the curve C parameterized by $\mathbf{r}(t) = \langle t\cos t, t\sin t, t\rangle$. This curve wraps counterclockwise around the cone $z^2 = x^2 + y^2$, as shown in the pictures below.



(a) (2 points) Show that C is smooth everywhere. (That is, show $\mathbf{r}'(t) \neq \mathbf{0}$ for any value of t.)

Solution: The parameterization $\mathbf{r}(t)$ is smooth since $\mathbf{r}'(t) = \langle \cos t - t\sin t, \sin t + t\cos t, 1\rangle \neq \mathbf{0}$ for any t (for one thing, the z-component is never zero). So C must be smooth, since it has a smooth parameterization.

(b) (3 points) Give an intuitive reason why the curvature of C should go to zero as the curve winds up the cone.

Solution: As C winds around the cone, it makes bigger and bigger sweeps. The radius of the osculating circle goes to infinity, and so the curvature must go to zero.

(c) (4 points) Compute $\kappa(0)$, the curvature of the curve C at $t = 0$. You may assume any of the formulas for curvature:

$$\kappa = \left|\frac{d\mathbf{T}}{ds}\right| = \frac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|} = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}.$$

Solution: With an eye toward using the last formula, let's compute a few derivatives:

$$\mathbf{r}'(t) = \langle \cos t - t\sin t, \sin t + t\cos t, 1\rangle$$
$$\mathbf{r}''(t) = \langle -2\sin t - t\cos t, 2\cos t - t\sin t, 0\rangle$$

Thus $\mathbf{r}'(0) = \langle 1,0,1\rangle$ and $\mathbf{r}''(0) = \langle 0,2,0\rangle$, and so

$$\mathbf{r}'(0) \times \mathbf{r}''(0) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 0 & 1 \\ 0 & 2 & 0 \end{vmatrix} = \langle -2, 0, 2\rangle.$$

Thus

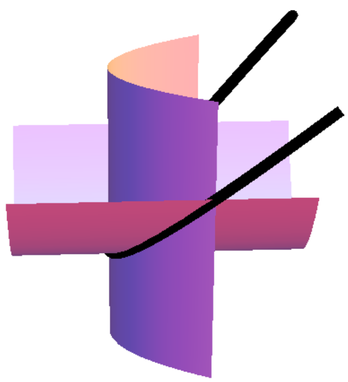
$$\kappa(0) = \frac{|\mathbf{r}'(0) \times \mathbf{r}''(0)|}{|\mathbf{r}'(0)|^3} = \frac{|\langle -2,0,2\rangle|}{|\langle 1,0,1\rangle|^3} = \frac{2\sqrt{2}}{(\sqrt{2})^3} = 1.$$

Thus $\boxed{\text{the curvature at the origin is } 1}$.

(d) (3 points) Find an equation for the osculating plane to C at the origin.

Solution: Our parameterization goes through the origin at $t = 0$. We can use $\mathbf{r}'(0) \times \mathbf{r}''(0)$ as the normal vector for the osculating plane, and we've already seen that $\mathbf{r}'(0) \times \mathbf{r}''(0) = \langle -2, 0, 2\rangle$. So the osculating plane is $\boxed{-2x + 2z = 0}$ or, more simply, $\boxed{z = x}$.

$\boxed{4}$ (8 points) Let C be the intersection of the surfaces $y = x^2$ and $z = x^2$, as shown in the pictures below.



(a) (5 points) Find a parameterization of C.

Solution: The curve lies on both surfaces, so we have $z = y = x^2$. Letting $x = t$, we have

$$\mathbf{r}(t) = \langle t, t^2, t^2\rangle.$$

(b) (3 points) Write down the integral that represents the distance along the curve C between the point $(1,1,1)$ and the point $(-1,1,1)$. You do **not** need to evaluate this integral!

Solution: The parameterization from part (a) goes through $(-1,1,1)$ at $t = -1$ and $(1,1,1)$ at $t = 1$. Since $\mathbf{r}'(t) = \langle 1, 2t, 2t\rangle$, we have

$$L = \int_{-1}^{1} \left|\mathbf{r}\,'(t)\right|\, dt = \int_{-1}^{1} \sqrt{1 + 8t^2}\, dt.$$

This is our answer.

$\boxed{5}$ (9 points) Consider the solid described by the inequalities

$$0 \le x \le 6 \qquad \text{and} \qquad 0 \le y^2 + z^2 \le 4.$$

The surface of this solid consists of three pieces: a cylinder, and two disks.

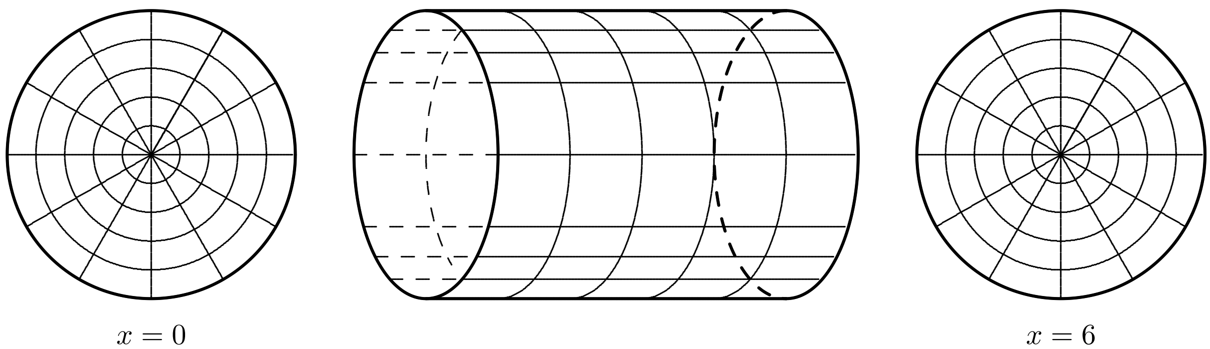
(a) (5 points) Find a parameterization of each piece of the surface. Give bounds on each parameter.

Solution: Let S_1 be the front disk (with $x = 6$), S_2 the back disk (with $x = 0$), and S_3 the cylinder. Then we have the following parameterizations:

$$\begin{array}{llll} S_1: & \mathbf{v}(r,\theta) = \langle 6, r\cos\theta, r\sin\theta\rangle, & 0 \le r \le 2, & 0 \le \theta \le 2\pi \\ S_2: & \mathbf{v}(r,\theta) = \langle 0, r\cos\theta, r\sin\theta\rangle, & 0 \le r \le 2, & 0 \le \theta \le 2\pi \\ S_3: & \mathbf{v}(x,\theta) = \langle x, 2\cos\theta, 2\sin\theta\rangle, & 0 \le x \le 6, & 0 \le \theta \le 2\pi \end{array}$$

(b) (4 points) Draw in the grid lines on the surfaces below corresponding to the parameterizations you found in part (a).

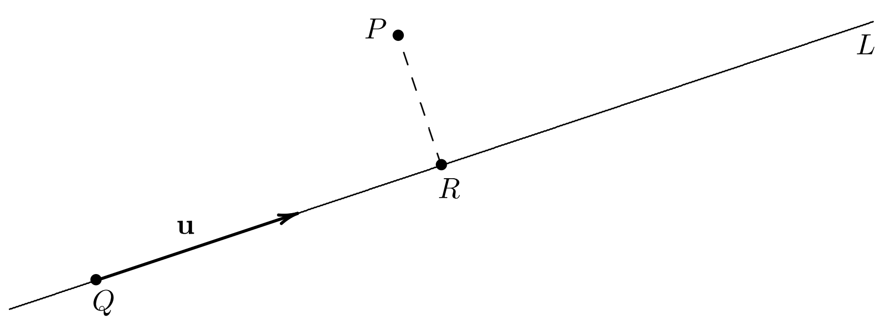
Solution:



6 (12 points)

(a) (4 points) Let L be the line given parametrically by $x = 4 + t$, $y = -1 - 2t$, $z = 5 + t$. Find the point on the line L which is closest to $(-2, 2, -1)$.

Solution: There are many ways to solve this problem. Let's start with what we know. L is given by the parametric vector equation $\mathbf{r}(t) = \langle 4, -1, 5\rangle + t\langle 1, -2, 1\rangle$. So, $Q(4, -1, 5)$ is a point on the line, and $\mathbf{u} = \langle 1, -2, 1\rangle$ is a vector parallel to the line. Let P be the point $(-2, 2, -1)$. We are looking for the point R on the line L which is closest to P. Here's a diagram:

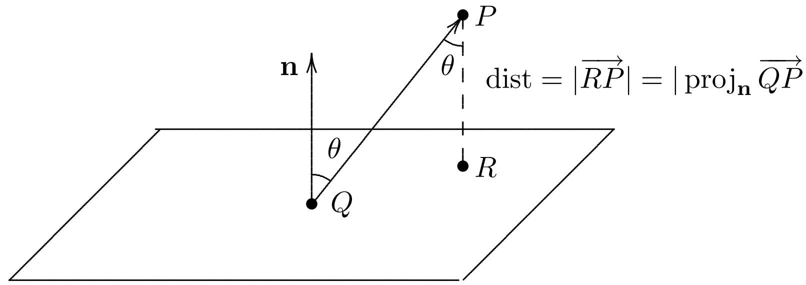


Here are two different approaches that both work well:

- Notice that $\text{proj}_{\mathbf{u}} \overrightarrow{QP}$ is equal to $\overrightarrow{QR}$. $\overrightarrow{QP} = \langle -6, 3, -6\rangle$, so the projection is simply $\frac{\overrightarrow{QP}\cdot\mathbf{u}}{\mathbf{u}\cdot\mathbf{u}}\mathbf{u} = \langle -3, 6, -3\rangle$. This means that R must be the point $\boxed{(1, 5, 2)}$.
- Alternatively, notice that $\overrightarrow{PR} \perp \mathbf{u}$, so $\overrightarrow{PR} \cdot \mathbf{u} = 0$. Since R is on the line L, it can be written as $(4 + t, -1 - 2t, 5 + t)$ for some t. If we can find t, then we will know what R is. We can now compute: $\overrightarrow{PR} = \langle 6+t, -3-2t, 6+t\rangle$, so $\overrightarrow{PR}\cdot\mathbf{u} = 1(6+t) - 2(-3-2t) + 1(6+t) = 18 + 6t$. This is equal to 0 when $t = -3$, which means that $\boxed{R = (1, 5, 2)}$ as before.

(b) (4 points) Find the point on the plane $2x-3y-z=-7$ which is closest to the point $(7,-2,-1)$.

Solution: Again, there are many different ways to solve this problem. Let's start with what we know. We have a point $P(7,-2,-1)$ and a plane whose normal vector is $\mathbf{n}=\langle 2,-3,-1\rangle$. We can also come up with a point on the plane by finding any (x,y,z) that satisfies the equation $2x-3y-z=-7$. For instance, let's use $Q(0,0,7)$. We want to find the point R on the plane which is closest to P. Here is a diagram.

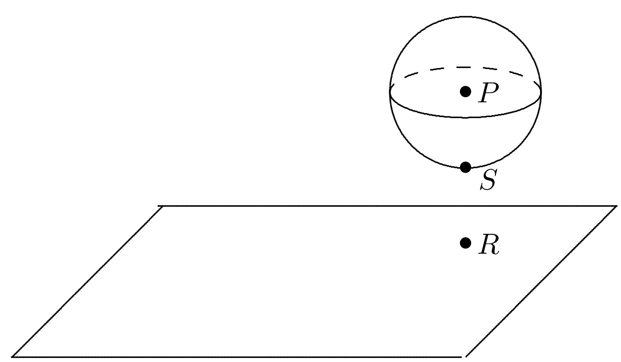


Here are two different approaches.

- Notice that $\operatorname{proj}_{\mathbf{n}}\overrightarrow{QP}$ is the same as $\overrightarrow{RP}$. Calculate $\operatorname{proj}_{\mathbf{n}}\overrightarrow{QP}=\langle 4,-6,-2\rangle$, so R is the point $\boxed{(3,4,1)}$.
- Alternatively, find the line L which passes through P and is parallel to $\mathbf{n}$. This line intersects the plane at R.
  Since the line passes through $P(7,-2,-1)$ and is parallel to $\mathbf{n}=\langle 2,-3,-1\rangle$, it can be described parametrically as $\langle 7,-2,-1\rangle+t\langle 2,-3,-1\rangle=\langle 7+2t,-2-3t,-1-t\rangle$.
  To find where the line intersects the plane, we use the fact that a point (x,y,z) is on the line if $x=7+2t$, $y=-2-3t$, $z=-1-t$ for some t, and this point is on the plane if $2x-3y-z=-7$. Plugging our expressions for x, y, and z into this second equation gives $2(7+2t)-3(-2-3t)-(-1-t)=-7$. Solving for t, we find $t=-2$, which corresponds to the point $x=7+2(-2)=3$, $y=-2-3(-2)=4$, $z=-1-(-2)=1$. That is, the point on the plane $2x-3y-z=-7$ which is closest to the point $(7,-2,-1)$ is $\boxed{(3,4,1)}$ as before.

(c) (4 points) Find the point on the sphere $(x-7)^2+(y+2)^2+(z+1)^2=16$ which is closest to the plane $2x-3y-z=-7$.

Solution: Notice that the sphere is centered at the point $(7,-2,-1)$, which is the point from the previous part, and its radius is 4. The plane is also the same as the previous part. So, here's a diagram, with R being the point we found in the previous part and S being the point we are looking for:



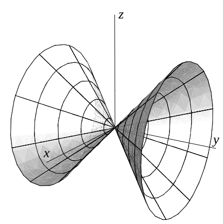
Notice that $\overrightarrow{PS}$ goes in the same direction as $\overrightarrow{PR}$ and has length 4 (the radius of the sphere). The unit vector in the direction of $\overrightarrow{PR}$ is $\frac{\overrightarrow{PR}}{|\overrightarrow{PR}|}=\frac{\langle -4,6,2\rangle}{2\sqrt{14}}=\frac{1}{\sqrt{14}}\langle -2,3,1\rangle$. The vector $\overrightarrow{PS}$

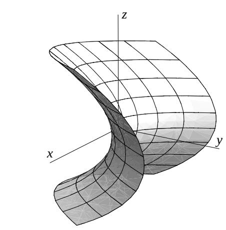
must be 4 times this, so $\overrightarrow{PS} = \frac{4}{\sqrt{14}}\langle -2, 3, 1\rangle = \left\langle -\frac{8}{\sqrt{14}}, \frac{12}{\sqrt{14}}, \frac{4}{\sqrt{14}} \right\rangle$. Therefore, the point S is $\boxed{\left(7 - \frac{8}{\sqrt{14}}, -2 + \frac{12}{\sqrt{14}}, -1 + \frac{4}{\sqrt{14}}\right)}$.

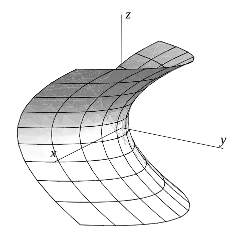
$\boxed{7}$ (10 points) Pick the picture that each equation describes, and mark your answers in the space indicated below.

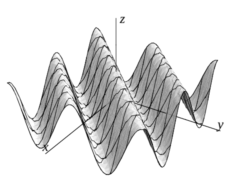
(a) $z = \cos(x - y)$ (b) $x^2 - y - z^2 = 0$ (c) $x^2 - y + z^2 = 1$

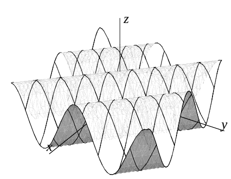
(d) $x^2 - y^2 + z^2 = 0$ (e) $x^2 - y^2 + z^2 = -1$

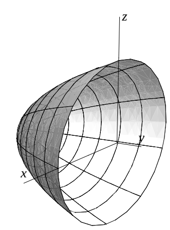

(A)

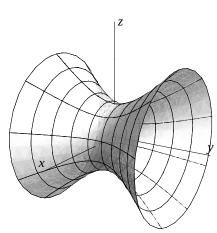

(B)

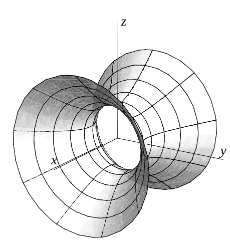

(C)

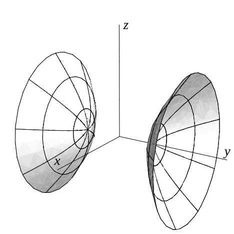

(D)


(E)


(F)


(G)


(H)


(I)

Solution: As usual, the key to understanding what a surface looks like is to look at its traces.

(a) The trace in $y = k$ of this surface is $z = \cos(x - k)$, which is a translate of the cosine curve. This matches either (D) or (E). To figure out which one, look at traces in $z = k$. To be really concrete, let's look at the trace in $z = 1$.

This is $\cos(x - y) = 1$, or $x - y = 0, \pm 2\pi, \pm 4\pi, \ldots$. This is a bunch of parallel lines $y = x, x \pm 2\pi, x \pm 4\pi, \ldots$. This matches (D).

(b) The traces in $x = k$ and $z = k$ are parabolas; the traces in $y = k$ are hyperbolas. This could match either (B) or (C).

To decide which, let's look more carefully at one trace, like the trace in $z = 0$. This is a parabola $y = x^2$ in the xy-plane, which opens toward the positive y direction. So, the correct picture is (B).

(c) The traces in $x = k$ and $z = k$ are both parabolas, and the traces in $y = k$ are ellipses. This matches (F), an elliptic paraboloid.

(d) The traces in $x = k$ and $z = k$ are both hyperbolas. The traces in $y = k$ are ellipses. This could match (A), (G), or (I). (Note that it could not match (H), because the traces in $x = k$ of (H) are ellipses.)

One way to see which is the right picture is just to look at the trace in $y = 0$: in (A), it's a point; in (G), it's an ellipse; in (I), it's nothing. The trace in $y = 0$ is $x^2 + z^2 = 0$, which describes a point. So, the right picture is (A).

(e) The traces in $x = k$ and $z = k$ are both hyperbolas, and the traces in $y = k$ are ellipses. Again, we can look at the trace in $y = 0$ to distinguish. In this case, the trace in $y = 0$ is $x^2 + z^2 = -1$, which describes nothing. So, the right picture is (I).

This problem was similar to the homework problems you did in §9.6, as well as #3 from the Spring 2008 practice exam and #15 from the review problems sheet.

$\boxed{8}$ (9 points)

(a) (2 points) Which one of the following is the same as $\phi = \frac{\pi}{6}$ in spherical coordinates?

(i) $z = \sqrt{x^2 + y^2}$ in Cartesian coordinates.

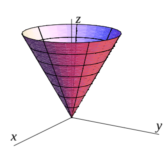
(ii) $z = 3r$ in cylindrical coordinates.

(iii) $z = \sqrt{r}$ in cylindrical coordinates.

(iv) $z^2 = 3(x^2 + y^2)$ in Cartesian coordinates.

(v) None of the above.

Solution: Here is a picture of $\phi = \frac{\pi}{6}$:



(It is not necessary to visualize to solve the problem.)

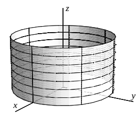
To convert $\phi = \frac{\pi}{6}$ to cylindrical coordinates, we use the fact that $\tan \phi = \frac{r}{z}$. Since $\tan \frac{\pi}{6} = \frac{1}{\sqrt{3}}$, the surface can be described in cylindrical coordinates as $\frac{r}{z} = \frac{1}{\sqrt{3}}$, or $z = \sqrt{3}r$. This eliminates (ii) and (iii).

To convert to Cartesian coordinates, we use the fact that $r = \sqrt{x^2 + y^2}$, so $z = \sqrt{3(x^2 + y^2)}$. This eliminates (i). (iv) looks like it may be the same, but in (iv), z is allowed to be negative, so (iv) actually describes a cone which opens both up and down.

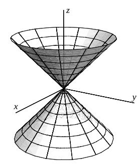
Therefore, the correct answer is $\boxed{\text{(v), none of the above}}$.

This problem was similar to #2 of the non-book problems on Homework 9 and #5 from the Spring 2008 practice exam.

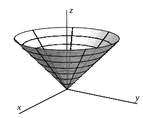
(b) (2 points) Which one of the following is a picture of the surface defined in cylindrical coordinates by $z = r$ and $0 \le r \le 1$?



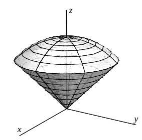
(i)



(ii)



(iii)



(iv)

Solution: $z = r$ can be written in Cartesian coordinates as $z = \sqrt{x^2 + y^2}$, which describes half of a cone. This matches (iii).

This problem was similar to #2 of the non-book problems on Homework 9 and #5 from the Spring 2008 practice exam.

(c) (2 points) Let $\mathcal{U}$ be the solid bounded below by $z = x^2 + y^2$ and above by $x^2 + y^2 + z^2 = 2$. Which one of the following is a description of $\mathcal{U}$?

(i) $r^2 \ge z \ge 2 - r^2$ in cylindrical coordinates.

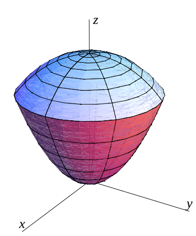
(ii) $\rho \le 2$, $\phi \ge \frac{\pi}{4}$ in spherical coordinates.

(iii) $r^2 \le z \le \sqrt{2 - r^2}$ in cylindrical coordinates.

(iv) $\sin\phi \le \rho \le 2$ in spherical coordinates.

(v) None of the above.

Solution: $z = x^2 + y^2$ describes an elliptic paraboloid with its tip at the origin, opening upward. $x^2 + y^2 + z^2 = 2$ describes a sphere of radius $\sqrt{2}$ with its center at the origin. Therefore, the solid in question looks like this:



Since $\mathcal{U}$ is bounded below by $z = x^2 + y^2$, it satisfies the inequality $z \ge x^2 + y^2$. Since it is bounded above by (the top half of) $x^2 + y^2 + z^2 = 2$, it also satisfies $z \le \sqrt{2 - (x^2 + y^2)}$. So, it is described by inequalities as $x^2 + y^2 \le z \le \sqrt{2 - (x^2 + y^2)}$.

In cylindrical coordinates, $x^2 + y^2 = r^2$, so the solid would be described as $r^2 \le z \le \sqrt{2 - r^2}$, which matches (iii).

This problem was similar to #3 of the non-book problems on Homework 9 and #8 on the review problems sheet.

(d) (3 points) Parameterize the surface described in spherical coordinates by $\theta = \phi$.

Solution: Remember that the goal of parameterizing a surface is to describe it (in Cartesian coordinates) using 2 variables.

We know we can describe this surface in spherical coordinates using just ρ and θ (since ϕ is always equal to θ). So, we just need to convert back to Cartesian coordinates, using $\phi = \theta$

$$\begin{aligned} x &= \rho \sin\theta \cos\theta \\ y &= \rho \sin\theta \sin\theta \\ z &= \rho \cos\theta \end{aligned}$$

This gives us our parameterization $\mathbf{r}(\rho, \theta) = \langle \rho \sin\theta \cos\theta, \rho \sin\theta \sin\theta, \rho \cos\theta \rangle$.

What you had to do in this problem was similar to what you did in the homework problem §10.5, #22 (but simpler).

$\boxed{9}$ (10 points) Let $A = (0, 0, 1)$ and $B = (0, 2, 3)$. Find the set of points $P(x, y, z)$ such that $\overrightarrow{AP}$ is orthogonal to $\overrightarrow{BP}$. Give a geometric description.

Solution: All we're told is that $\overrightarrow{AP}$ and $\overrightarrow{BP}$ are perpendicular, which means that $\overrightarrow{AP} \cdot \overrightarrow{BP} = 0$. We can calculate the two vectors $\overrightarrow{AP} = \langle x-0, y-0, z-1\rangle = \langle x, y, z-1\rangle$ and $\overrightarrow{BP} = \langle x-0, y-2, z-3\rangle = \langle x, y-2, z-3\rangle$. Taking their dot product, we get

$$0 = \overrightarrow{AP} \cdot \overrightarrow{BP} = \langle x, y, z-1\rangle \cdot \langle x, y-2, z-3\rangle = x^2 + y^2 - 2y + z^2 - 4z + 3.$$

From here we complete the square:

$$0 + 1 + 4 = x^2 + \left(y^2 - 2y + 1\right) + \left(z^2 - 4z + 4\right) + 3, \qquad \text{or} \qquad 2 = x^2 + (y-1)^2 + (z-2)^2.$$

This is a sphere of radius $\sqrt{2}$ centered at the point $(0, 1, 2)$.

$\boxed{10}$ (10 points) Suppose $\mathbf{a}$ and $\mathbf{b}$ are vectors about which we know: $|\mathbf{a}| = 3$, $|\mathbf{b}| = 2$, and $\mathbf{a} \times \mathbf{b} = \langle 1, -5, 1\rangle$. Find the following quantities, if possible. If you cannot find a particular value because there is not enough information, indicate this.

(a) (2 points) $\mathbf{a} \cdot \mathbf{b}$

Solution: We know that $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}|\ |\mathbf{b}| \cos(\theta)$, but does this help? We're told $|\mathbf{a}| = 3$ and $|\mathbf{b}| = 2$. What about $\cos(\theta)$? We can find $\sin(\theta)$, since $|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}|\ |\mathbf{b}| \sin(\theta)$:

$$\sin(\theta) = \frac{|\mathbf{a} \times \mathbf{b}|}{|\mathbf{a}|\ |\mathbf{b}|} = \frac{|\langle 1, -5, 1\rangle|}{(3)(2)} = \frac{\sqrt{27}}{6} = \frac{3\sqrt{3}}{6} = \frac{\sqrt{3}}{2}.$$

So $\cos(\theta)$ is $\frac{1}{2}$ or possibly $-\frac{1}{2}$. Both of these are possible, so we $\boxed{\text{cannot determine } \mathbf{a} \cdot \mathbf{b}}$.

(b) (2 points) $|\mathbf{a} \cdot \mathbf{b}|$

Solution: This is $|\mathbf{a} \cdot \mathbf{b}| = |\mathbf{a}|\ |\mathbf{b}|\ |\cos(\theta)|$. From the answer to part (a), we can calculate $|\cos(\theta)| = \left|\pm\frac{1}{2}\right| = \frac{1}{2}$, so $|\mathbf{a} \cdot \mathbf{b}| = |\mathbf{a}|\ |\mathbf{b}|\ |\cos(\theta)| = (3)(2)(\frac{1}{2}) = \boxed{3}$.

(c) (2 points) The acute angle between a line in the direction of $\mathbf{a}$ and a line in the direction of $\mathbf{b}$

Solution: In part (a) we found that $\sin(\theta) = \frac{\sqrt{3}}{2}$. This means either that θ, the angle between the vectors $\mathbf{a}$ and $\mathbf{b}$, is either $\frac{\pi}{3}$ or $\frac{2\pi}{3}$. These two angles are the angles between the two lines in our question, so the acute angle is $\boxed{\frac{\pi}{3}}$ or $\boxed{60°}$.

(d) (2 points) $|\text{proj}_{\mathbf{a}}\,\mathbf{b}|$

Solution: We clearly cannot find $\text{proj}_{\mathbf{a}}\,\mathbf{b} = \frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|^2}\mathbf{a}$, since that depends on the direction of $\mathbf{a}$. But we're asked for the magnitude of this, which is

$$|\text{proj}_{\mathbf{a}}\,\mathbf{b}| = \frac{|\mathbf{a}\cdot\mathbf{b}|}{|\mathbf{a}|^2}\,|\mathbf{a}| = \frac{|\mathbf{a}\cdot\mathbf{b}|}{|\mathbf{a}|} = \frac{3}{3} = 1.$$

(e) (2 points) An equation of the plane through the origin parallel to both $\mathbf{a}$ and $\mathbf{b}$

Solution: The usual way to find this vector is to compute the normal as $\mathbf{a}\times\mathbf{b}$. This is given to us: $\mathbf{a}\times\mathbf{b} = \langle 1, -5, 1\rangle$. The plane through the origin $(0,0,0)$ is then $\boxed{\langle 1, -5, 1\rangle \cdot \langle x, y, z\rangle = 0}$ or $\boxed{x - 5y + z = 0}$.