Math 21a | First Midterm Exam | Spring, 2009

Your Name

Your Signature

Instructions:

- Please begin by printing and signing your name in the boxes above and by checking your section in the box to the right.
- You are allowed 2 hours (120 minutes) for this exam. Please pace yourself accordingly.
- You may not use any calculator, notes, or other assistance on this exam.
- In order to receive full credit, you must show your work and carefully justify your answers. The correct answer without any work will receive little or no credit.
- If you need more room, use the backs of the pages and indicate to the grader that you have done so.
- Please write neatly. Illegible answers will be assumed to be incorrect.
- Raise your hand if you have a question.
- Good luck!

☐	MWF 9	John Hall
☐	MWF 10	Janet Chen
☐	MWF 11	Peter Garfield
☐	MWF 12	Peter Garfield
☐	TTh 10	Jun Yin

Problem	Total Points	Score
1	8	
2	12	
3	12	
4	8	
5	9	

Problem	Total Points	Score
6	12	
7	10	
8	9	
9	10	
10	10	
Total	100	

1 (8 points)

(a) (4 points) Find an equation for the plane containing the three points $P(3,3,1)$, $Q(2,-1,0)$, and $R(-1,-3,1)$.

(b) (4 points) Are the four points P, Q, R, and $S(7,4,-1)$ coplanar? (Here P, Q, and R are the points from part (a).) Justify your answer.

2 (12 points)

(a) (4 points) Find an equation for the plane given by the parameterization

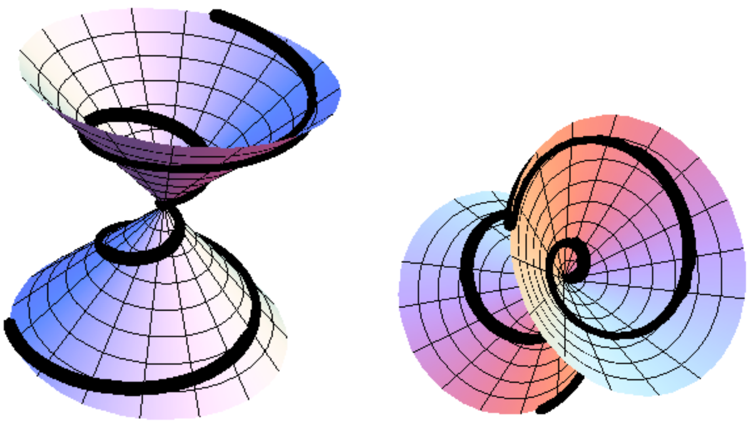
$$\mathbf{r}(u, v) = \langle 3 + 2u, 5 - u + v, 2u + 3v \rangle .$$

(b) (3 points) Suppose the curve C is parameterized with respect to arc length by $\mathbf{r}(t)$ (that is, this parameterization has $|\mathbf{r}'(t)| = 1$ for all t). What is the distance along C between $\mathbf{r}(3)$ and $\mathbf{r}(10)$?

(c) (2 points) Suppose the traces of a quadric surface are parabolas ($x = k$), parabolas ($y = k$), and hyperbolas ($z = k$). What quadric surface is this? Explain your reasoning.

(d) (3 points) An ant is standing on the surface $z = x^3 - 3xy + e^{xy}$ at the point $(1, 0, 2)$. If the ant walks East (that is, in the positive x direction), is he moving up or down? Explain your reasoning.

3 (12 points) Consider the curve C parameterized by $\mathbf{r}(t) = \langle t\cos t, t\sin t, t\rangle$. This curve wraps counterclockwise around the cone $z^2 = x^2 + y^2$, as shown in the pictures below.



(a) (2 points) Show that C is smooth everywhere. (That is, show $\mathbf{r}'(t) \neq \mathbf{0}$ for any value of t.)

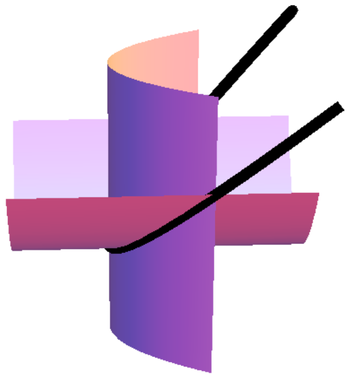
(b) (3 points) Give an intuitive reason why the curvature of C should go to zero as the curve winds up the cone.

(c) (4 points) Compute $\kappa(0)$, the curvature of the curve C at $t = 0$. You may assume any of the formulas for curvature:

$$\kappa = \left|\frac{d\mathbf{T}}{ds}\right| = \frac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|} = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}.$$

(d) (3 points) Find an equation for the osculating plane to C at the origin.

4 (8 points) Let C be the intersection of the surfaces $y = x^2$ and $z = x^2$, as shown in the pictures below.



(a) (5 points) Find a parameterization of C.

(b) (3 points) Write down the integral that represents the distance along the curve C between the point $(1, 1, 1)$ and the point $(-1, 1, 1)$. You do **not** need to evaluate this integral!

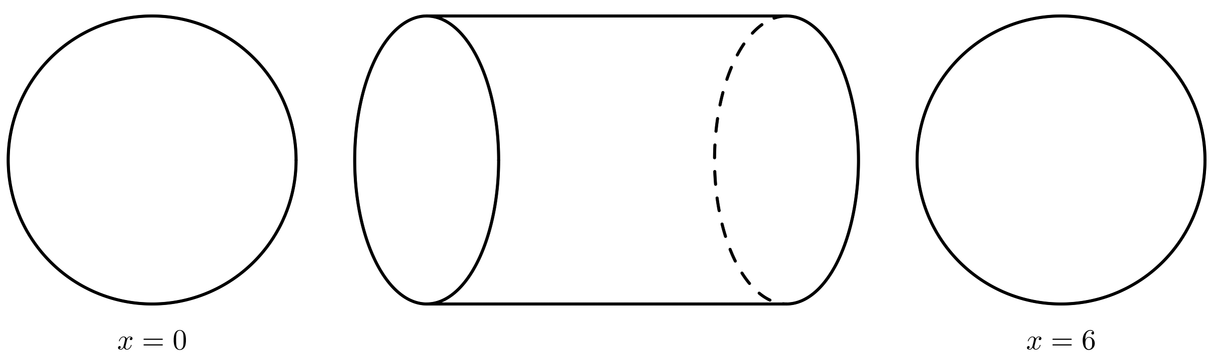
5 (9 points) Consider the solid described by the inequalities

$$0 \leq x \leq 6 \qquad \text{and} \qquad 0 \leq y^2 + z^2 \leq 4.$$

The surface of this solid consists of three pieces: a cylinder, and two disks.

(a) (5 points) Find a parameterization of each piece of the surface. Give bounds on each parameter.

(b) (4 points) Draw in the grid lines on the surfaces below corresponding to the parameterizations you found in part (a).



6 (12 points)

(a) (4 points) Let L be the line given parametrically by $x = 4 + t$, $y = -1 - 2t$, $z = 5 + t$. Find the point on the line L which is closest to $(-2, 2, -1)$.

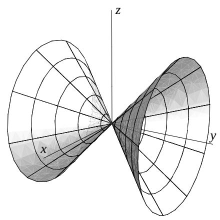
(b) (4 points) Find the point on the plane $2x-3y-z = -7$ which is closest to the point $(7, -2, -1)$.

(c) (4 points) Find the point on the sphere $(x - 7)^2 + (y + 2)^2 + (z + 1)^2 = 16$ which is closest to the plane $2x - 3y - z = -7$.

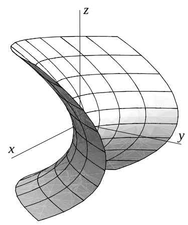
7 (10 points) Pick the picture that each equation describes, and mark your answers in the space indicated below.

(a) $z = \cos(x - y)$ (b) $x^2 - y - z^2 = 0$ (c) $x^2 - y + z^2 = 1$

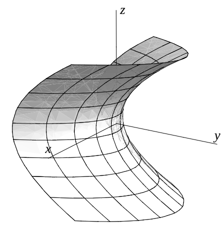
(d) $x^2 - y^2 + z^2 = 0$ (e) $x^2 - y^2 + z^2 = -1$



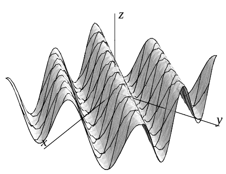
(A)



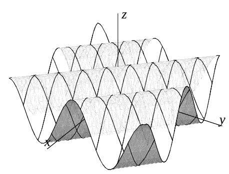
(B)



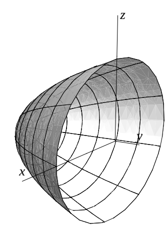
(C)



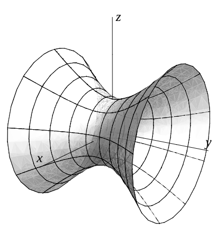
(D)



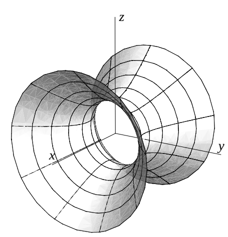
(E)



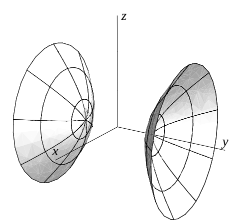
(F)



(G)



(H)



(I)

Mark your answers here:

(a) __________ (b) __________ (c) __________

(d) __________ (e) __________

8 (9 points)

(a) (2 points) Which one of the following is the same as $\phi = \frac{\pi}{6}$ in spherical coordinates?

(i) $z = \sqrt{x^2 + y^2}$ in Cartesian coordinates.

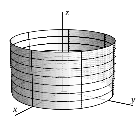
(ii) $z = 3r$ in cylindrical coordinates.

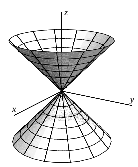
(iii) $z = \sqrt{r}$ in cylindrical coordinates.

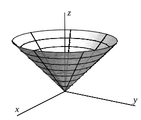
(iv) $z^2 = 3(x^2 + y^2)$ in Cartesian coordinates.

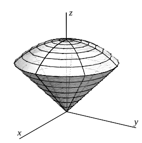
(v) None of the above.

(b) (2 points) Which one of the following is a picture of the surface defined in cylindrical coordinates by $z = r$ and $0 \leq r \leq 1$?


(i)


(ii)


(iii)


(iv)

(c) (2 points) Let $\mathcal{U}$ be the solid bounded below by $z = x^2 + y^2$ and above by $x^2 + y^2 + z^2 = 2$. Which one of the following is a description of $\mathcal{U}$?

(i) $r^2 \geq z \geq 2 - r^2$ in cylindrical coordinates.

(ii) $\rho \leq 2$, $\phi \geq \frac{\pi}{4}$ in spherical coordinates.

(iii) $r^2 \leq z \leq \sqrt{2 - r^2}$ in cylindrical coordinates.

(iv) $\sin\phi \leq \rho \leq 2$ in spherical coordinates.

(v) None of the above.

(d) (3 points) Parameterize the surface described in spherical coordinates by $\theta = \phi$.

9 (10 points) Let $A = (0, 0, 1)$ and $B = (0, 2, 3)$. Find the set of points $P(x, y, z)$ such that $\overrightarrow{AP}$ is orthogonal to $\overrightarrow{BP}$. Give a geometric description.

10 (10 points) Suppose $\mathbf{a}$ and $\mathbf{b}$ are vectors about which we know: $|\mathbf{a}| = 3$, $|\mathbf{b}| = 2$, and $\mathbf{a} \times \mathbf{b} = \langle 1, -5, 1 \rangle$. Find the following quantities, if possible. If you cannot find a particular value because there is not enough information, indicate this.

(a) (2 points) $\mathbf{a} \cdot \mathbf{b}$

(b) (2 points) $|\mathbf{a} \cdot \mathbf{b}|$

(c) (2 points) The acute angle between a line in the direction of $\mathbf{a}$ and a line in the direction of $\mathbf{b}$

(d) (2 points) $|\text{proj}_{\mathbf{a}}\, \mathbf{b}|$

(e) (2 points) An equation of the plane through the origin parallel to both $\mathbf{a}$ and $\mathbf{b}$