Math 21a

Second Midterm Exam

Spring, 2009

Your Name

Your Signature

Instructions:

- Please begin by printing and signing your name in the boxes above and by checking your section in the box to the right.
- You are allowed 2 hours (120 minutes) for this exam. Please pace yourself accordingly.
- You may not use any calculator, notes, or other assistance on this exam.
- In order to receive full credit, you must show your work and carefully justify your answers. The correct answer without any work will receive little or no credit.
- If you need more room, use the backs of the pages and indicate to the grader that you have done so.
- Please write neatly. Illegible answers will be assumed to be incorrect.
- Raise your hand if you have a question.
- Good luck!

☐	MWF 9	John Hall
☐	MWF 10	Janet Chen
☐	MWF 11	Peter Garfield
☐	MWF 12	Peter Garfield
☐	TTh 10	Jun Yin

Problem	Total Points	Score
1	10	
2	10	
3	11	
4	10	
5	10	

Problem	Total Points	Score
6	12	
7	10	
8	5	
9	12	
10	10	
Total	100	

1 (10 points) Find all critical points of $f(x,y) = x^2y - x^2 - 2y^2$, and classify each as a local minimum, local maximum, or saddle point.

2 (10 points) Consider the double integral

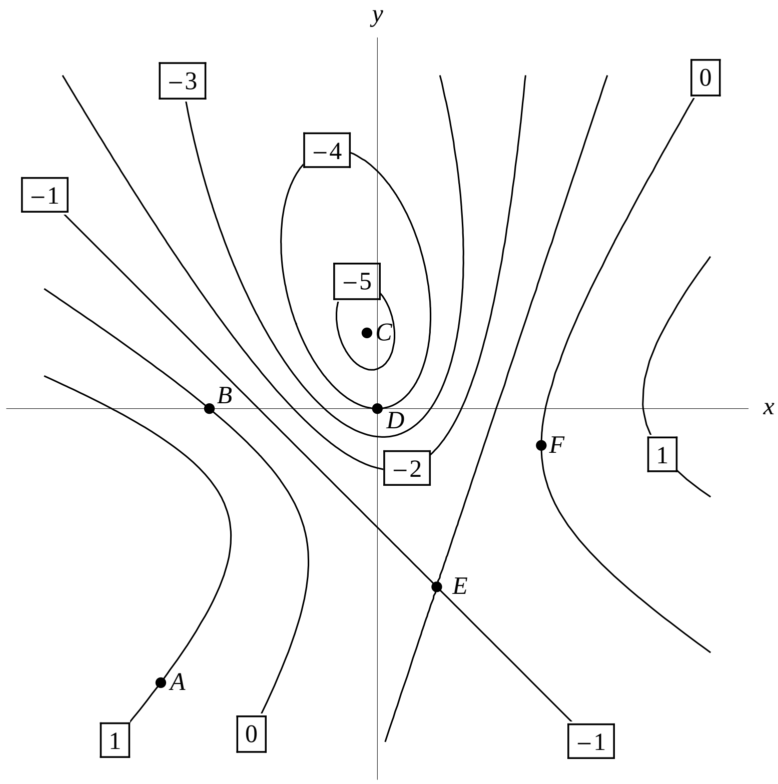
$$\int_0^2 \int_{y/2}^1 y e^{x^3} \, dx \, dy$$

(a) (3 points) Sketch the region of integration. Label all curves and all points of intersection and shade in the region.

(b) (4 points) Rewrite the integral using the order $dy\,dx$.

(c) (3 points) Compute the integral.

3 (11 points) Here is the level set diagram (contour map) of a function $f(x, y)$. The value of f on each level set is indicated. Two of the six labeled points are critical points of f.



(a) (4 points) Which two points are critical points of f?

A B C D E F

Classify each critical point as a local minimum, local maximum, or saddle point.

- Point _______ is a ________________________________.
- Point _______ is a ________________________________.

(b) (4 points) Two of the following four points have the property that $\frac{\partial f}{\partial x}$ is 0 at the point. Which two?

B D E F

(c) (3 points) Decide whether the following directional derivatives are greater than 0, less than 0, or equal to 0. Circle the appropriate phrase.

i. $D_{\mathbf{u}}f(A)$, where $\mathbf{u} = \left\langle \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}} \right\rangle$.

greater than 0 less than 0 equal to 0

ii. $D_{\mathbf{u}}f(C)$, where $\mathbf{u} = \left\langle \frac{3}{5}, \frac{4}{5} \right\rangle$.

greater than 0 less than 0 equal to 0

iii. $D_{\mathbf{u}}f(D)$, where $\mathbf{u} = \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle$.

greater than 0 less than 0 equal to 0

4 (10 points) Let E be the solid consisting of all points (x, y, z) inside both the sphere $x^2+y^2+z^2 = 4$ and the cylinder $x^2 + y^2 = 1$. Find the volume of E.

5 (10 points) Consider the surface given by $z = f(x, y)$, where $f(x, y) = x^4 + y^4 - 4xy + 10$.

(a) (3 points) At the point where $x = -1$ and $y = 2$, in which direction(s) is the height of the surface above the xy-plane *decreasing* most rapidly? Give your answer(s) in the form of a unit vector $\mathbf{u} = \langle a, b \rangle$.

(b) (3 points) At the point where $x = -1$ and $y = 2$, in which direction(s) is the height of the surface not changing at all? Give your answer(s) in the form of a unit vector $\mathbf{u} = \langle a, b \rangle$.

(c) (4 points) Use linear approximation to estimate $f(-0.8, 2.1)$.

6 (12 points) Let $f(x, y) = x^2 - y^2$.

(a) (5 points) Let C be the level curve of f through the point $(4, 3)$. Find the tangent line to C at the point $(4, 3)$.

(b) (3 points) Let $\mathbf{u}$ be the unit vector in the direction of $\langle 1, 2\rangle$. Find the directional derivative $D_{\mathbf{u}}f$ at the point (x, y).

(c) (4 points) Let $\mathbf{u}$ be the unit vector in the direction of $\langle 1, 2\rangle$. In the region $x^2 + y^2 \leq 4$, what is the maximum value of $D_{\mathbf{u}}f$?

7 (10 points) Let D be the region bounded by the parabola $x = y^2$ and the line $x + y = 2$.

(a) (4 points) Sketch the region D. Label all curves and all points of intersection and shade in the region D.

(b) (3 points) Write $\iint_D f(x, y)\, dA$ as an iterated integral (or a sum of iterated integrals) using the order $dx\, dy$.

(c) (3 points) Write $\iint_D f(x, y)\, dA$ as an iterated integral (or a sum of iterated integrals) using the order $dy\, dx$.

8 (5 points) Indicate whether each statement is true or false. No explanations are required.

(a) **T** **F** If (a, b) is a critical point of $f(x, y)$ such that $f_{xx}f_{yy} - f_{xy}^2 > 0$ at (a, b) and $f_{yy}(a, b) > 0$, then (a, b) must be a local minimum of $f(x, y)$. (You may assume that f and all of its derivatives are continuous.)

(b) **T** **F** The function $f(x, y) = x^2 + 2y^2$ attains an absolute minimum on $3x + 5y = 1$.

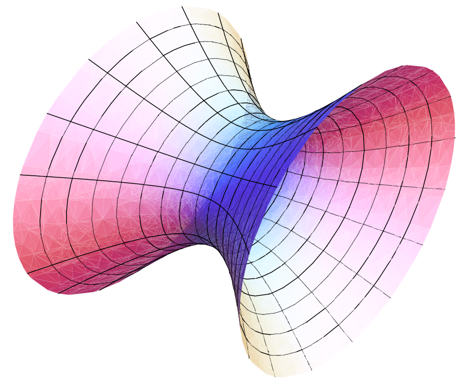
(c) **T** **F** The function $f(x, y) = x^2 + 2y^2$ attains an absolute maximum on $3x + 5y = 1$.

(d) **T** **F** The function $f(x, y) = 3x + 5y$ attains an absolute minimum on $x^2 + 2y^2 = 1$.

(e) **T** **F** The function $f(x, y) = 3x + 5y$ attains an absolute maximum on $x^2 + 2y^2 = 1$.

9 (12 points) Consider the hyperboloid

$$2x^2 - y^2 + z^2 = 2$$



(a) (6 points) Find every point on the hyperboloid where the tangent plane is parallel to the plane $x - y - z = 0$.

(b) (6 points) Find the point (or points) on the hyperboloid closest to the origin.

10 (10 points) Use a double integral to find the area bounded by one petal of the rosette

$$r = \sin(3\theta).$$

You may find it useful to recall that

$$\sin^2(x) = \frac{1}{2}\Big(1 - \cos(2x)\Big)$$

and

$$\cos^2(x) = \frac{1}{2}\Big(1 + \cos(2x)\Big).$$