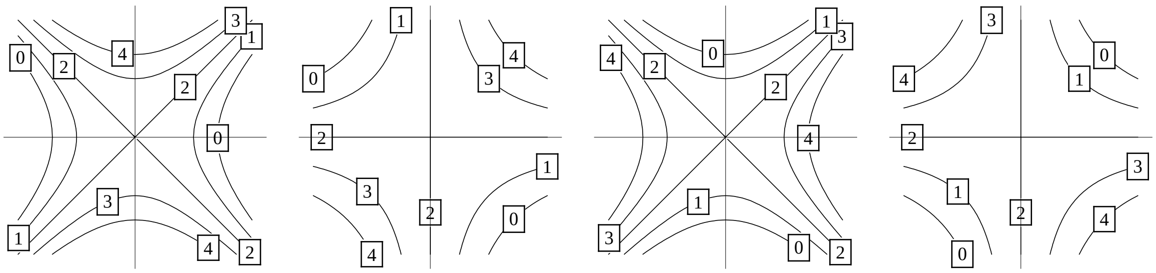
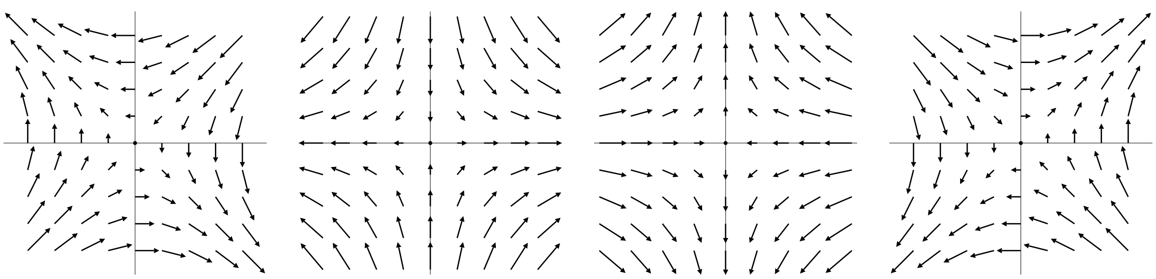
Extra Problems on Sections 13.1 - 13.4

1. The first row shows level set diagrams (contour maps) of four functions f. The second row shows the vector fields ∇f for the same four functions. Match each level set diagram with the corresponding vector field. Explain your reasoning.



(I) (II) (III) (IV)



(a) (b) (c) (d)

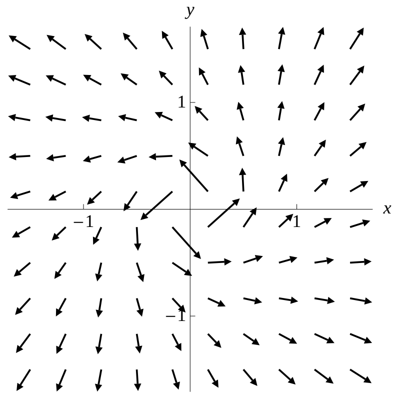
2. Let $\vec{F}$ be the vector field on $\mathbb{R}^2$ defined by $\vec{F}(x,y) = \langle 2xy, x^2 - \sin y \rangle$.

   (a) Let (a,b) be any point in $\mathbb{R}^2$, and let C be the straight-line path from $(0,0)$ to (a,b). Parameterize C, and use your parameterization to compute $\int_C \vec{F} \cdot d\vec{r}$.

   (b) Show that $\vec{F}$ is a gradient vector field.

   (c) Find a function f such that $\nabla f = \vec{F}$. (How does your answer relate to your answer from (a)?)

3. Consider the vector field $\vec{F}(x,y) = \langle P(x,y), Q(x,y) \rangle = \left\langle -\frac{y}{x^2+y^2} + x, \frac{x}{x^2+y^2} + y \right\rangle$ from #4(b) on the worksheet "The Fundamental Theorem for Line Integrals; Gradient Vector Fields". In class, we did not decide conclusively whether this vector field was conservative. Now, show that it is in fact **not** conservative.



4. In each part, evaluate the line integral $\int_C \vec{F} \cdot d\vec{r}$, using any method you like.

(a) $\vec{F}(x, y) = \langle y, -x \rangle$ and C is the ellipse $4x^2 + 9y^2 = 16$, oriented counterclockwise.

(b) $\vec{F}(x, y) = \left\langle \tan(x^3), e^{y^2} \right\rangle$ and C is the circle $(x-2)^2 + y^2 = 1$, oriented counterclockwise.

(c) $\vec{F}(x, y) = \langle y, x \rangle$ and C is the path parameterized by $\langle \sin 5t + \cos 3t, \sin 4t + \sin^2 2t + \cos 3t + \cos^2 t \rangle$, $0 \le t \le \pi$.

(d) $\vec{F}(x, y) = \langle e^{x^3}, y^2 \rangle$ and C is the right half of the ellipse $9x^2 + y^2 = 1$, oriented counterclockwise.

(e) $\vec{F}(x, y) = \langle 3x^2 y \sin(x^3) + y + 2, -\cos(x^3) \rangle$ and C is the top half of the circle $x^2 + y^2 = 1$, oriented counterclockwise.