6/27/2002 CROSS PRODUCT,LINES,PLANES (Section 9.4-9.5) S-Math 21a

This is part 3 (of 3) of the homework which is due July 2 at the beginning of class. More problems to this lecture can be found on pages 674-675 and 683-685 in the book.

SUMMARY.

- $v \times w = (v_2 w_3 - v_3 w_1, v_3 w_1 - v_1 w_3, v_1 w_2 - v_2 w_1)$ **cross product**.
- $|v \times w| = |v||w| \sin(\phi)$, **angle** ϕ between vectors.
- $v \times w$ is **orthogonal** both to v and to w.
- $u \cdot (v \times w)$ **triple product**, volume of parallelepiped spanned by u, v, w.
- $n \cdot \mathbf{x} = ax + by + cz = d$ equation for **plane**.
- $r(t, s) = r_0 + tv + sw$ parametric equation for a **plane**.
- $r(t) = r_0 + tv$ parametric equation for a **line**.

1) (4 points) Find a unit vector orthogonal to both $(1, -1, 1)$ and $(0, 4, 4)$.

2) (4 points) Find the equation of the form $ax + by + cz = d$ for a plane which passes through the three points $(1, 0, 0), (1, 1, 0)$ and $(1, 1, 1)$.

3) (4 points) Find the parametric equation for the line which is the intersection of $x + y + z = 1$ and $x + z = 0$.

4) (4 points) Find a parametric equation (of the form $r(t, s) = r_0 + tv + sw$) of the plane $x + 2y + z = 3$.

5) (4 points) Find the distance between the point $(1, 1, 1)$ and the plane $x + y - z = 2$.

CHALLENGE PROBLEM:

Find a general formula for the volume of a tetrahedron with edges P, Q, R, S.

Hint. Find first a formula for the area of one of its triangular faces, and then a formula for the distance from the fourth point to that face.

SUPER CHALLENGE PROBLEM:

The coordinates for the edges of a cube in 4D are the 16 points $(\pm 1, \pm 1, \pm 1, \pm 1)$. Find the angle between the big diagonal connecting $(1, 1, 1, 1)$ with $(-1, -1, -1, -1)$ and the "middle diagonal" in one of 3D faces connecting $(1, 1, 1, 1)$ with $(-1, -1, -1, 1)$.