7/10/2002 ARC LENGTH, PARAMETRIC SURFACES (10.3/10.5) S-Math 21a

This is part 2 (of 2) of the homework for the third week. It is due July 16 at the beginning of class. More problems to this lecture can be found on pages 723-725 and 740-742 in the book.

SUMMARY.

- $|r'(t)| = \sqrt{x'(t)^2 + y'(t)^2 + z'(t)^2}$ **speed** of $r(t) = (x(t), y(t), z(t))$.
- $\int_a^b \sqrt{x'(t)^2 + y'(t)^2 + z'(t)^2} dt$ **arc length**.
- $|r'(t) \times r''(t)|/|r'(t)|^2$ **curvature**.
- $r(u, v) = (x(u, v), y(u, v), z(u, v))$ **parametric surface**.

EXAMPLES:

- $r(t) = P + t\vec{v}$ **line** through P.
- $r(t) = (\cos(t), 0, \sin(t))$ **circle**
- $r(u, v) = P + u\vec{u} + v\vec{v}$ **plane**
- $r(u, v) = (\cos(u)\sin(v), \sin(u)\sin(v), \cos(v))$ **sphere**
- $r(u, v) = (\cos(u), \sin(u), v)$ **cylinder**

1) (6 points, compare problems 2 in 10.3) Find the length of the curve $r(t) = (t^2, \sin(t) - t\cos(t), \cos(t) + t\sin(t))$, $0 \leq t \leq \pi$.

2) (6 points, compare problem 4 in 10.3) Find the length of the curve $r(t) = (t^2, 2t, log(t))$, $t \in [0, e]$. (Note: log denotes the natural logarithm which satisfies $\log(e) = 1$.)

3) (6 points, compare problem 18 in 10.3) Find the curvature of $r(t) = (e^t\cos(t), e^t\sin(t), t)$ at the point $(1, 0, 0)$.

4) (6 points, compare problem 18 in 10.4) Find a parametrisation of the lower half of the ellipsoid $2x^2 + 4y^2 + z^2 = 1$.

5) (6 points, compare problem 32 a in 10.4) Find a parametrisation of the **torus** which is obtained as the set of points which have distance 1 from the circle $(x(t), y(t), z(t)) = (2\cos(t), 2\sin(t), 0)$.

Hint: Keep $u = t$ as one of the parameters.

CHALLENGE PROBLEM:

Sketch the surface $r(u, v) = (2 + 2v\cos(\pi u))\cos(2\pi u), (2 + 2v\cos(\pi u))\sin(2\pi u), v\sin(\pi u))$.

SUPER CHALLENGE PROBLEM:

The **torus** is obtained by bending and gluing the ends of a cylinder together. The **Klein bottle** is obtained in the same way, however, the ends are put together with opposite directions. This can not be achieved without self-intersection. Take one end of the tube, bend it, enter the tube first to match the ends in the opposit direction as for the torus. Can you find a parametrisation $r(u, v)$ for this surface?