7/16/2002 PARTIAL DERIVATIVES (Section 11.1/11.4) **S-Math 21a**

This is part 1 (of 3) of the weekly homework. It is due July 23 at the beginning of class. More problems to this lecture can be found on pages 756-759 and 788-789 in the book.

SUMMARY.

- $\frac{\partial f}{\partial x} f(x,y,z) = f_x(x,y,z)$ **partial derivative**.
- $\nabla f(x,y,z) = \text{grad}(f(x,y,z)) = (f_x(x,y,z), f_y(x,y,z), f_z(x,y,z))$ **gradient**
- $f_{xy} = f_{yx}$ for smooth functions.
- $L(x,y) = f(x_0,y_0) + f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0)$ **linear approximation**. Allows estimating $f(x,y)$ by $L(x,y)$ near $f(x_0,y_0)$.

1) (4 points) Sketch the graph and the contour map of the function $f(x,y) = x^2 + 9y^2$. Find the gradient of f at the point $(1,1)$ and draw it.

2) (4 points) The gas law for a fixed mass m of an ideal gas at absolute temperature T, pressure P and volume V is $PV = mRT$, where R is the gas constant. Show that

$$\frac{\partial P}{\partial V}\frac{\partial V}{\partial V}\frac{\partial T}{\partial P} = -1$$

3) (4 points) The ellipsoid $4x^2 + 2y^2 + z^2 = 16$ intersects the plane $y = 2$ in an ellipse. Find parametric equations for the tangent line to this ellipse at the point $(1,2,2)$.

4) (4 points) Verify that $f(x,t) = e^{-rt} f(x+ct)$ satisfies the equation $f_t(x,t) = cf_x(x,t) - rf(x,t)$. This equation is an example of a partial differential equation. It is called the **advection equation**.

5) (4 points) Find the linear approximation $L(x,y)$ of the function $f(x,y) = \sqrt{20 - x^2 - 7y^2}$ at $(2,1)$ and use it to estimate $f(1.95, 1.08)$.

CHALLENGE PROBLEM:

Is there a non-constant function $f(x,y)$ with the property that $f_x(x,y)f_y(x,y) = 0$ at all points?

SUPER CHALLENGE PROBLEM:

Given two functions $g(x,y) = x^2$ and $h(x,y) = y^3 + x^2$. Is there a function $f(x,y)$ such that $\nabla f(x,y) = (g(x,y), h(x,y))$?