7/17/2002 CHAIN RULE, DIRECTIONAL DERIVATIVE (Section 11.5/11.6) S-Math 21a

This is part 2 (of 3) of the weekly homework. It is due July 23 at the beginning of class. More problems to this lecture can be found on pages 796-798 and 808-810 in the book.

SUMMARY.

- $f(x, y, z)$ function of three variables, $r(t)$ **curve**, $\frac{d}{dt}f(r(t)) = \nabla f(r(t)) \cdot r'(t)$ **chain rule**.
- u unit vector, $D_u f(x, y, z) = \nabla f(x, y, z) \cdot u$ **directional derivative**.
- $n = \nabla f(x_0, y_0, z_0)$ is orthogonal to $f(x, y, z) = c$, where $c = f(x_0, y_0, z_0)$.
- $n \cdot (x, y, z) = d = n \cdot (x_0, y_0, z_0$ tangent plane to $f(x, y, z) = c$ at point (x_0, y_0, z_0).

1) (4 points) Let $r(t) = (\cos(t), \sin(t), t)$ and $T(x, y, z) = x^2 + y^2 + 2z^2$. Find $\frac{d}{dt}T(r(t))$ at $t = 0$ in two different ways. First by differentiating the function $g(t) = T(r(t))$ and second, using the chain rule.

2) (4 points) Find the equation for the tangent plane at the surface $x^2 + 2y^2 - z^2 = 1$ at the point $(2, 1, 0)$.

3) (4 points) Find the directions in which the directional derivative of $f(x, y) = x^2 + \sin(xy)$ at the point $(1, 0)$ has the value 1.

4) (4 points) Verify that the surface $z^2 = x^2 + y^2$ and $x^2 + y^2 + z^2 = 1$ are orthogonal at every point of intersection.

5) (4 points) Show that the x-,y- and z- intercepts of any tangent plane to the surface $\sqrt{x} + \sqrt{y} + \sqrt{z} = \sqrt{c}$ is constant.

CHALLENGE PROBLEM:

The partial derivatives of the function $f(x, y) = (xy)^{1/3}$ exists at every point but the directional derivatives in all other directions don't exist. What is going on?

SUPER CHALLENGE PROBLEM:

Extend the notion of tangent plane to 5-dimensional hypersurfaces $f(x, y, z, u, v, w) = c$ in 6-dimensional space.