7/18/2002 (CONSTRAINED) EXTREMA (Section 11.7/11.8) **S-Math 21a**

This is part 3 (of 3) of the weekly homework. It is due July 23 at the beginning of class. More problems to this lecture can be found on pages 818-820 and 827-829 in the book.

SUMMARY.

- $\nabla f(x,y) = (0,0,0)$ **critical point**.
- $H(x,y) = \begin{bmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{bmatrix}$ **Hessian**. $D = \det(H(x,y))$ **determinant** $f_{xy}f_{yy} - f_{xy}^2$.
- $D > 0, f_{xx} < 0$ **local maximum**, $D > 0,\ f_{xx} > 0$ **local minimum**, $D < 0$ **saddle point**.
- Extremize $f(x,y,z)$ under the constraint $g(x,y,z) = c$: $\nabla f(x,y,z) = \lambda \nabla g(x,y,z)$ with **Lagrange multiplier** λ.

1) (4 points) Find all the extrema of the function $f(x,y) = xy^3 - yx^3$ and determine whether they are maxima, minima or saddle points.

2) (4 points) Find the extrema of the function $f(x,y) = e^{-x^2-y^2}(x^2+2y^2)$ inside the disk $x^2+y^2 < 4$.

3) (4 points) Find the extrema of the function $f(x,y) = e^{-x^2-y^2}(x^2+2y^2)$ on the circle $g(x,y) = x^2+y^2 = 4$ using the method of Lagrange multipliers. What is the maximum of $f(x,y)$ inside the disk $x^2+y^2 \leq 4$?

4) (4 points) Find the points on the surface $xy^2z^3 = 2$ that are closest to the origin.

5) (4 points) The entropy of a coin is defined as the $-\log(p) - \log(q)$, where p is the probability that head appears and q is the probability that tail appears For which (p,q) $(p+q=1)$ is the entropy maximal?

CHALLENGE PROBLEM:

What does it mean that the Lagrange multiplier λ is zero in a constrained optimization problem?

SUPER CHALLENGE PROBLEM:

How would the criterium for local maxima, local minima for functions $f(x,y,z)$ of three variables look like?