

This is part 1 (of 2) of the weekly homework. It is due July 30 at the beginning of class. More problems to this lecture can be found on pages 847-848, 853-854, 861-862 and 867-868 in the book.

SUMMARY.

- $\int \int_R f \, dA = \int_a^b \int_c^d f(x, y) \, dydx$ **double integral** over rectangle.
- $\int \int_R f \, dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x, y) \, dydx$ double integral over **type I region**.
- $\int \int_R f \, dA = \int_a^b \int_{h_1(y)}^{h_2(y)} f(x, y) \, dx dy$ double integral over **type II region**.
- $A(R) = \int \int_R 1 \, dA$ **area** of A .
- $\frac{1}{A(R)} \int \int_R f \, dA$ **average value** of f on R .
- $\int \int_R f(x, y) \, dx dy = \int_\alpha^\beta \int_a^b f(r \cos(\theta), r \sin(\theta)) \, r dr d\theta$
integral in polar coordinates.

- 1) (6 points) Calculate the iterated integral $\int_1^3 \int_0^1 (1 + 4xy) \, dx dy$.
- 2) (6 points) Find the volume of the solid lying under the paraboloid $z = x^2 + y^2$ and above the rectangle $R = [-2, 2] \times [-3, 3]$.
- 3) (6 points) Calculate the iterated integral $\int_0^1 \int_y^{e^y} \sqrt{x} \, dx dy$.
- 4) (6 points) Evaluate the following iterated integral by converting into polar coordinates

$$\int_0^2 \int_0^{\sqrt{2x-x^2}} \sqrt{x^2 + y^2} \, dy dx .$$

- 5) (6 points) Find the average value of $f(x, y) = x^2 + y^2$ on the annulus $1 \leq |(x, y)| \leq 2$.

CHALLENGE PROBLEM:



The integral $\int_0^1 \arccos(\sqrt{x}) \, dx$ can be written as a double integral $\int_0^1 \int_0^{\arccos(\sqrt{x})} dy dx$. Calculate this integral by switching the order of integration.

SUPER CHALLENGE PROBLEM:



Calculate $\int \int_{\mathbf{R}^2} e^{-x^2-y^2} \, dx dy$.

Hint. The function $f(x) = \int e^{-x^2}$ is known to have no anti-derivative which can be expressed with "known functions" like exp, log, sin etc. You can nevertheless integrate the above integral using polar coordinates.