

7/24/2002 TRIPLE INTEGRALS, SURFACE AREA (Section 12.6-12.9) S-Math 21a

This is part 2 (of 2) of the weekly homework. It is due July 30 at the beginning of class. More problems to this lecture can be found on pages 881-882, 890-892 and 898-900 in the book.

SUMMARY.

- $r(u, v)$ **parametric surface** defined on $[a, b] \times [c, d]$ has area $\int_a^b \int_c^d |r_u(u, v) \times r_v(u, v)| \, dudv$.
- $\iiint_E f \, dV = \int_a^b \int_c^d \int_e^f f(x, y, z) \, dzdydx$ **triple integral**.
- $V(E) = \iiint_E 1 \, dV$ **volume**.
- $\frac{1}{V(E)} \iiint_E f \, dV$ **average value** of f over E .
- $\iiint_E f \, dV = \int_c^d \int_\alpha^\beta \int_a^b f(\rho \sin(\phi) \cos(\theta), \rho \sin(\phi) \sin(\theta), \rho \cos(\phi)) \rho^2 \sin(\phi) \, d\rho d\theta d\phi$ integral over **spherical wedge**.

- 1) (6 points) Find the exact area of the surface $z = 1 + 2x + 3y + 4y^2$ with $1 \leq x \leq 4, 0 \leq y \leq 1$.
- 2) (6 points) A torus is parametrized by $r(t) = (2 \cos(u) + \cos(u) \cos(v), 2 \sin(u) + \sin(u) \cos(v), \sin(v))$. Find the area of the torus.
- 3) (6 points) Evaluate the following iterated integral $\int_0^3 \int_0^1 \int_0^{\sqrt{1-z^2}} z e^y \, dx dz dy$.
- 4) (6 points) Find the volume of the solid bounded by the paraboloid $z = x^2 + y^2$ and $z = 18 - x^2 - y^2$.
- 5) (6 points) Find the mass and center of mass of a solid hemi sphere of radius 1 if the density at any point is proportional to its distance from its base. (In other words, calculate $\iiint_E z \, dx dy dz$, where E is the part of the unit sphere with $z \geq 0$.)

CHALLENGE PROBLEM:



Find the volume of the intersection of two cylinders $y^2 + z^2 = 1$ and $x^2 + z^2 = 1$. You have a picture on page 882 in the text.

Hint. Look what happens, when you cut the body at a fixed z value and calculate the area of this section.

SUPER CHALLENGE PROBLEM:



Use a quadruple integral to find the volume of the hyper-sphere $x^2 + y^2 + z^2 + w^2 = r^2$ in \mathbf{R}^4 .

Hint. If you slice up the hyper-sphere at w , you get a sphere of radius $\sqrt{r^2 - w^2}$. Integrate the volume of the sphere from $w = -r$ to $w = r$ using substitution.

Can you find a formula for the volume of the n dimensional hyper-sphere?