7/31/2002 LINE INTEGRALS (Section 13.2) **S-Math 21a**

This is part 2 (of 3) of the weekly homework. It is due August 6 at the beginning of class. More problems to this lecture can be found on pages 933-935 in the book.

SUMMARY.

- $\int_C F \cdot dr = \int_a^b F(r(t)) \cdot r'(t)dt$ **line integral** of F along curve r.

1) (4 points) Let C be the right half of the circle $x^2 + y^2 = 16$ and $F(x, y) = (x, y^4)$. Calculate the line integral $\int_C F \cdot dr$.

2) (4 points) Let C be the curve $r(t) = (\cos(t), \sin(t), t)$ for $t \in [0, 1]$ and $F(x, y, z) = (y, x, 5)$. Calculate the line integral $\int_C F \cdot dr$.

3) (4 points) Calculate the line integral $F(x, y) = (x/\sqrt{x^2 + y^2}, y/\sqrt{x^2 + y^2})$ along the parabola $y = 1 + x^2$ from $(-1, 2)$ to $(1, 2)$.

4) 

(4 points) A 160 lb = 80 kg tourist walks up the tower in Provincetown. How much work did he do after climbing the 80 meters.

5) (4 points) The gravitational force of the sun onto the earth is $F(x, y, z) = (x, y, z)mMG/|r|^3$, where $M = 210^{3}0kg$ is the mass of the sun, $m = 610^{24}kg$ is the mass of the earth and $G = 710^{-11}Nm^2/kg^2$ is the gravitational constant. What is work done by the gravitational field when the earth moves from aphelion distance 1.5210^8km from the sun to the perihelion distance 1.4710^8km?

CHALLENGE PROBLEM:

Let $F(x, y, z, w) = (x, y, z, w)$. Calculate the line integral along the curve connecting $(0, 0, 0, 0)$ with $(1, 1, 1, 1)$.

SUPER CHALLENGE PROBLEM:

Does the line integral concept still make sense with a different dot product $v \cdot w = g_1 v_1 w_1 + g_2 v_2 w_2 + g_3 v_3 w_3$?