8/01/2002 FTL, APPLICATIONS (13.3) **S-Math 21a**

This is part 3 (of 3) of the weekly homework. It is due August 6 at the beginning of class. More problems to this lecture can be found on pages 943-945 in the book.

SUMMARY.

- $\int_C \nabla f \; dr = f(r(b)) - f(r(a))$ **fundamental theorem of line integrals**
- $F = \nabla f \Leftrightarrow$ if and only if line integrals don't depend on path.

1) (4 points) Calculate the line integral $\int_C 2x\sin(y)dx + (x^2\cos(y) - 3y^2)dy$ along a straight line from $(-1, 0)$ to $(5, 1)$.

Hint. You can do this elegantly using the fundamental theorem of line integrals.

2) (4 points) Let $F(x, y) = (-y/(x^2 + y^2), x/(x^2 + y^2))$. Let $C : r(t) = (\cos(t), \sin(t)), t \in [0, 2\pi]$ be the unit circle. a) Calculate $\int_C F \cdot dr$.
b) Let $f(x, y) = \arctan(y/x)$. Show that $\nabla f = F$.
c) Why do a) and b) not contradict each other?

3) (4 points) Let $F = \nabla f$ and $f(x, y) = \sin(x - 2y)$. Find curves C_1 and C_2 that are not closed and satisfy the equation $\int_{C_1} F \cdot dr = 0$ and $\int_{C_2} F \cdot dr = 1$.

4) (4 points) Calculate the line integral of $F(x, y, z) = (-y, x, z^2)$
along the path $r(t) = (\cos(17t)\sin(t), \sin(2t)\cos(5t), \sin(23t))$ $t \in [0, 2\pi]$.

5) (4 points) Show that the line integral $\int_C ydx + xdy + xyzdz$ is not independent of the path.
Hint. If $F(x, y, z) = (P, Q, R)$ is independent of the path, then $P_y = Q_x, P_z = R_x, Q_z = R_y$.

CHALLENGE PROBLEM:

Consider a O shaped pipe which is filled only on the right side with water. A wooden ball falls in the air and is dragged up in the water. Why does this perpetuum mobile not work?

SUPER CHALLENGE PROBLEM:

What is wrong with the Escher picture which describes a stair in which a ball always falls down? This figures defines force fields which are not conservative. (Look for the figure with "Escher Upstairs Downstairs" in google.)