This is part 2 (of 3) of the weekly homework. It is due Aug 13 in the mailbox of Jon. More problems to this lecture can be found on pages 970-971 and 976-977 in the book.

SUMMARY.

- $\mathrm{curl}(P, Q, R) = (R_y - Q_x, P_z - R_x, Q_x - P_y)$ **curl**.

- $\mathrm{curl}\nabla f = (0, 0, 0)$.

- $\int\int_S f\,dS = \int_D f(r(u, v))|r_u \times r_v|\,dA$ **surface integral**.

- $\int\int_S F \cdot dS = \int_D F(r(u, v)) \cdot (r_u \times r_v)\,dA$ **flux integral**.

- $\int\int_S \mathrm{curl}F \cdot dS = \int_C F \cdot dr$ **Stokes theorem**.

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1) (4 points) Evaluate $\int\int_S yz\,dS$, where S is the surface with parametric equation $x = uv, y = u + v, z = u - v, u^2 + v^2 \leq 1$.

2) (4 points) Evaluate the flux integral $\int\int_S \mathrm{curl}(F) \cdot dS$ for $F(x, y, z) = (xy, yz, zx)$, where S is the part of the paraboloid $z = 4 - x^2 - y^2$ that lies above the square $[0, 1] \times [0, 1]$ and has upward orientation.

3) (4 points) Evaluate the same flux integral as in the previous question but using Stokes theorem.

4) (4 points) Use Stokes theorem to evaluate $\int_C F \cdot dr$, where $F(x, y, z) = (x^2y, x^3/3, xy)$ and C is the curve of intersection of the hyperbolic paraboloid $z = y^2 - x^2$ and the cylinder $x^2 + y^2 = 1$, oriented counterclockwise as viewed from above.

5) (4 points) If S is a sphere and F is a smooth vector field, show that $\int\int_S \mathrm{curl}(F) \cdot dS = 0$.

CHALLENGE PROBLEM:

Use Stokes theorem to show that $\int_C (f\nabla g + g\nabla f) \cdot dr = 0$.

SUPER CHALLENGE PROBLEM:

Try to figure out, how Stokes theorem would look like in higher dimensions: in four dimensions, it is useful in special relativity.

Start: In dimension d, the curl is a field $\mathrm{curl}(F)_{ij} = \partial_{x_j}F_i - \partial_{x_i}F_j$ with $\binom{d}{2}$ components. In 4 dimensions, it has 6 components.

In d dimensions, a surface element in the $i - j$ plane is written as dS_{ij}. The flux integral of the curl of F through S is defined as $\int\int \mathrm{curl}(F) \cdot dS$, where the dot product is $\sum_{i<j} \mathrm{curl}(F)_{ij}dS_{ij}$. If S is given by a map X from a planar domain R to R^d, $U = \partial_u X$ and $V = \partial_v X$ are tangent vectors to that plane and $dS_{ij}(u, v) = (U_iV_j - U_jV_i)\,dudv$.

Try to make sense of it, formulate Stokes theorem.