8/08/2002 DIVERGENCE THEOREM (Section 13.8-13.9) **S-Math 21a**

This is part 3 (of 3) of the weekly homework. It is due August 13 in the mailbox of Jon. More problems to this lecture can be found on pages 983-984 and 987-988 in the book.

SUMMARY.

- $\mathrm{div}(F)(x, y, z) = F_x + F_y + F_z$ **divergence**. Is a scalar field.
- $\int\int\int_E \mathrm{div}(F)\, dV = \int\int_S F \cdot dS$ **divergence theorem**.

1) (4 points) Verify that the divergence theorem is true for the vector field $F(x, y, z) = (3x, xy, 2xz)$ on the unit cube $[0, 1] \times [0, 1] \times [0, 1]$.

2) (4 points) Verify that the divergence theorem is true for the vector field $F(x, y, z) = (xy, yz, zx)$ on the solid cylinder $x^2 + y^2 \leq 1$, $0 \leq z \leq 1$.

3) (4 points) Use the divergence theorem to calculate the flux of $F(x, y, z) = (x^3, y^3, z^3)$ through the sphere $x^2 + y^2 + z^2 = 1$.

4) (4 points) Verify that $\mathrm{div}(E) = 0$ away from the origin if E is the electric field $E(x) = \epsilon Q x/|x|^3$.

5) (4 points) Find $\int\int\int_S F \cdot dS$, where $F(x, y, z) = (x, y, z)$ and S is the outwardly oriented surface obtained by removing the cube $[1, 2] \times [1, 2] \times [1, 2]$ from the cube $[0, 2] \times [0, 2] \times [0, 2]$.

CHALLENGE PROBLEM:

We show that Green's theorem in the plane is equivalent to Gauss theorem in the plane:

a) Let $G(x, y)$ be the vector field orthogonal to $F(x, y)$. Show that $\mathrm{div}(G) = \mathrm{curl}(F)$.

b) Show that the line integral $\int_C F\, dr$ along a curve C is the same as the flux integral $\int_C G \cdot dn$.

SUPER CHALLENGE PROBLEM:

Formulate the divergence theorem in arbitrary dimensions.