

Completion of square.

Reminder: The quadratic equation $x^2 + bx + c = 0$ is solved by adding $(b/2)^2 - c$ on both sides:

$$\begin{aligned} x^2 + bx + c &= 0 \\ x^2 + bx + (b/2)^2 &= (b/2)^2 - c \\ (x + b/2)^2 &= (b/2)^2 - c \\ x + b/2 &= \sqrt{(b/2)^2 - c} \\ x &= \pm\sqrt{(b/2)^2 - c} - b/2 . \end{aligned}$$

This method is called **completion of the square**.

One can use this to determine the center and radius of circle or spheres.

Example: $x^2 + 5x + y^2 - 2y = 1$ is equivalent to $(x + 5/2)^2 - 25/4 + (y - 1)^2 - 1 = 1$ or $(x - 5/2)^2 + (y - 1)^2 = (5/2)^2$. Therefore, the equation is a circle with center $(5/2, 1)$ and radius $5/2$.

Now its your turn: Find the center and radius of the sphere $x^2 + 6x + y^2 - 4y + z^2 - 2z = 4$.

1) Completion of square for x :

2) Completion of square for y :

3) Completion of square for z :

Result:

The equation is therefore a sphere of radius centered at the point