

This is part 2 (of 3) of the homework which is due July 8 at the beginning of class.

SUMMARY.

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| <ul style="list-style-type: none"> <li>• <math>f(x, y)</math> <b>function of two variables</b>,<br/><math>g(x, y, z)</math> function of three variables.</li> <li>• <math>\{f(x, y) = c\}</math> describes <b>curve</b> called<br/><b>contour curve</b>.</li> <li>• <math>\{g(x, y, z) = d\}</math> describes <b>surface</b> called<br/><b>contour surface</b>.</li> <li>• <math>\{(x, y, z = f(x, y))\}</math> <b>graph</b> of function<br/><math>f(x, y)</math>, describes a surface.</li> <li>• <b>trace</b>: intersection of graph with coordi-<br/>nate planes.</li> <li>• <b>intercepts</b>: intersection of graph with<br/>coordinate axes.</li> </ul> | <p><b>Quadrics:</b></p> <ul style="list-style-type: none"> <li>• <math>x^2 + y^2 + z^2 = 1</math> <b>sphere</b>.</li> <li>• <math>x^2 + y^2 - z^2 = 1</math> <b>one-sheeted<br/>hyperboloid</b>.</li> <li>• <math>x^2 + y^2 - z^2 = -1</math> <b>two-sheeted<br/>hyperboloid</b>.</li> <li>• <math>x^2 - y^2 - z^2 = 0</math> <b>cone</b>.</li> <li>• <math>x^2 + y^2 = z</math> <b>paraboloid</b>.</li> <li>• <math>x^2 + y^2 = 1</math> <b>cylinder</b>.</li> </ul> |
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## Homework Problems

- 1) (4 points) Let  $f(x, y) = x^2 - y^2$ .
    - a) (1) Write the graph of  $f$  as a contour surface  $g(x, y, z) = 0$ .
    - b) (1) Find the equations for the three traces of  $f$ , the intersections with the coordinate planes.
    - c) (2) Sketch the graph of  $f$ .
  
  - 2) (4 points) Consider the surface  $z^2 - x^2 - 2x - y = 0$ . Draw the three traces. What surface is it?
  
  - 3) (4 points) Surfaces satisfying the implicit equation  $x^k + y^k = z^k$  with integer  $k$  are called **Fermat** surfaces. In this exercise, we don't want you to compute, but to draw and visualize. Traces help you to get the picture right.
    - a) Sketch the Fermat surface for  $k = 2$  with traces.
    - b) Sketch the Fermat surface for  $k = 4$  with traces.
  
  - 4) (4 points)
    - a) (1) Sketch the graph of the function  $f(x, y) = 1/(x^2 + 4y^2)$ .
    - b) (1) Sketch the graph of the function  $g(x, y) = |x| + |y|$ .
    - c) (1) Sketch some contour curves  $f(x, y) = c$  of  $f$ .
    - d) (1) Sketch some contour curves  $f(x, y) = c$  of  $g$ .
  
  - 5) (4 points) Verify that the line  $\vec{r}(t) = (1, 3, 2) + t(1, 2, 1)$  is contained in the surface  $y = z^2 - x^2$ .
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## Challenge Problems

(Solutions to these problems are **not** turned in with the homework.)

- 1) Find as many curves as you can which are obtained by intersecting two quadrics. To get full credit for this problem you have to provide at least three different curves.
- 2) How would you visualize the graph of a function  $f(x, y, z)$  of three variables? How would you describe the set  $\{(x, y, z, u) \mid g(x, y, z, u) = d\}$ , where  $g$  is a function of 4 variables and  $d$  is a constant? Take the example of the four-dimensional sphere  $x^2 + y^2 + z^2 + u^2 = 1$ .