This is part 3 (of 3) of the homework which is due July 8 at the beginning of class.

SUMMARY:

- $x = r\cos(\theta)$
  $y = r\sin(\theta)$
  $z = z$
  cylindrical coordinates
- $x = \rho\sin(\phi)\cos(\theta)$
  $y = \rho\sin(\phi)\sin(\theta)$
  $z = \rho\cos(\phi)$
  spherical coordinates
- $\vec{r}(u,v) = (x(u,v), y(u,v), z(u,v))$ defines a **parametric surface**.

EXAMPLES:

- $r = 1$, **cylinder**.
- $\rho = 1$, **sphere**.
- $r = z$, **cone**
- $\vec{r}(u,v) = P + u\vec{u} + v\vec{v}$ **plane**
- $\vec{r}(u,v) = (\cos(u)\sin(v), \sin(u)\sin(v), \cos(v))$ **sphere**
- $\vec{r}(u,v) = (\cos(u), \sin(u), v)$ **cylinder**
- $\vec{r}(u,v) = (u, v, f(u,v))$ **graph** of f

Homework Problems

1) (4 points)
   a) What is the equation for the surface $x^2 + y^2 - 5x = z^2$ in cylindrical coordinates?
   b) Describe in words or draw a sketch of the surface whose equation is $\rho = \sin(\phi)$ in spherical coordinates (ρ, θ, ϕ).

2) (4 points) Plot the surface with the parametrization $\vec{r}(u,v) = (v^2\cos(u), v^2\sin(u), v)$, where $u \in [0, 2\pi]$ and $v \in \mathbf{R}$.

3) (4 points) Find a parametrization for the triangle in space which has the vertices $(3,4,1)$,$(1,2,1)$ and $(0,3,4)$.

4) (4 points) Find two different parametrisations of the lower half of the ellipsoid $2x^2+4y^2+z^2 = 1$.

5) (4 points) Find a parametrisation of the **torus** which is obtained as the set of points which have distance 1 from the circle $(x(t), y(t), z(t)) = (2\cos(t), 2\sin(t), 0)$. Hint: Keep $u = t$ as one of the parameters.

Challenge Problems

(Solutions to these problems are **not** turned in with the homework.)

1) Try to graph without computer the surface $r = f(\theta,\phi) = (2 + \sin(3\theta))(2 + \cos(2\phi))$ (It is a **graph** in spherical coordinates (r, ϕ, θ).)

Hint. Do it in stages. First graph $r = 2$ (the sphere), then $r = (2 + \sin(3\theta))$, then draw a sketch of the final surface.

2) How would you design analogues of spherical or cylindrical coordinates in 4 dimensions?

3) Sketch the surface $r(u, v) = (2 + 2v\cos(\pi u))\cos(2\pi u), (2 + 2v\cos(\pi u))\sin(2\pi u), v\sin(\pi u))$.

4) The **torus** is obtained by bending and gluing the ends of a cylinder together. The **Klein bottle** is obtained in the same way, however, the ends are put together with opposite directions. This can not be achieved without self-intersection. Take one end of the tube, bend it, enter the tube first to match the ends in the opposit direction as for the torus. Can you find a parametrisation $r(u, v)$ for this surface? On the handout for this lecture, you find a parametrization of the same surface but which looks different. The idea is to have a parametrization which produces the "bottle".