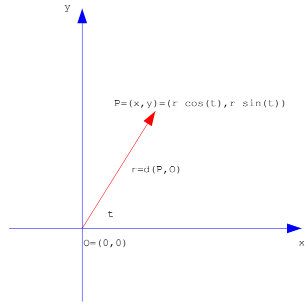
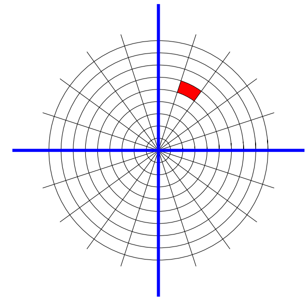
Lecture 6: 7/3/2003, CYLINDRICAL/SPHERICAL **Maths21a, O. Knill**

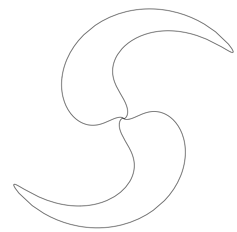
POLAR COORDINATES. A point (x, y) in the plane has **polar coordinates** $r = \sqrt{x^2 + y^2}, \theta = \arctan(y/x)$ (the later gives θ only modulo $\pi = 180$). The **radius** r is assumed to be nonnegative and the **angle** is $\theta \in [0, 2\pi]$. From r and θ, we recover $x = r\cos(\theta)$, $y = r\sin(\theta)$.





POLAR CURVES. A general polar curve is given in Polar coordinates as $t \mapsto (r(t), \theta(t))$. In rectangular coordinates the curve is $\vec{r}(t) = (x(t), y(t)) = (r(t)\cos(\theta(t)), r(t)\sin(\theta(t)))$.

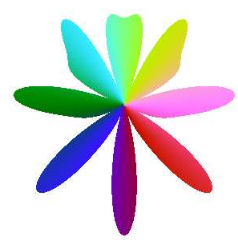
EXAMPLE. The polar curve $(r(t), \theta(t)) = (\sin(t)^4, t - \cos(4t))$ is shown to the right.



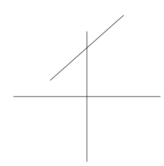
POLAR GRAPHS. Curves are graphs in polar coordinates are called **polar graphs** defined by a function $r(t) \geq 0$

$$\vec{r}(t) = (r(t)\cos(t), r(t)\sin(t)) .$$

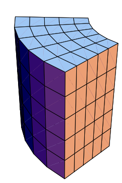
EXAMPLE. The polar graph defined by $r(\theta) = \cos(3\theta)$ is called the **trifoil**. It belongs to the class of **roses** $r(t) = |\cos(nt)|$.



EXAMPLE. The **line** $y = 2x + 3$ is in polar coordinates described by $r\sin(t) = 2r\cos(t + 3$, $\theta(t) = t$. The line is a polar graph: solving for $r(t)$ gives $r(\theta) = 3/(\sin(\theta - 2\cos(\theta))$.



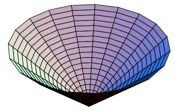
CYLINDRICAL COORDINATES. Cylindrical coordinates in space use polar coordinates in the x-y plane and leave the z coordinate. The relation between the coordinates is:



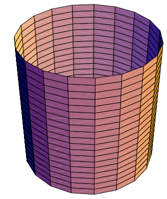
$(x, y, z) = (r\cos(\theta), r\sin(\theta), z)$	$(r, \theta, z) = (\sqrt{x^2 + y^2}, \arctan(y/x), z)$

SURFACES IN CYLINDRICAL COORDINATES. Similar, as $g(x, y, z) = 0$ defines a surface in rectangular coordinates, $g(r, \theta, z) = 0$ defines a surface in cylindrical coordinates.

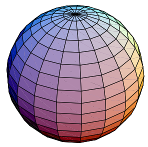
$r = z$, a **cone**



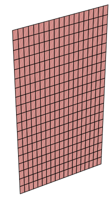
$r = 1$, a **cylinder**



$r^2 + z^2 = 1$ **sphere**



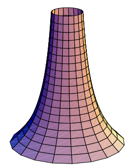
$\theta = \pi/2$, **half plane**

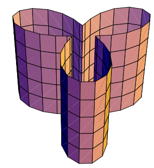


CYLINDRICAL GRAPHS. A surface $z = f(r, \theta)$ is called a **cylindrical graph**.

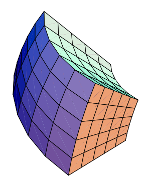
EXAMPLE. $r(\theta, z) = h(z)$ is a surface of revolution.

EXAMPLE. $r(\theta, z) = h(\theta)$ is a generalized cylinder.





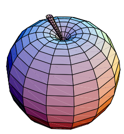
SPHERICAL COORDINATES. Spherical coordinates use two angles θ, ϕ as well as the distance ρ from the origin. The θ angle is as in cylindrical coordinates, ϕ is the angle between (x, y, z) and the $(0, 0, 1)$. The relation between the coordinates is:

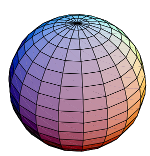


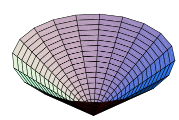
$$(x, y, z) = (\rho\cos(\theta)\sin(\phi), \rho\sin(\theta)\sin(\phi), \rho\cos(\phi)) \quad (\rho, \theta, \phi) = (\sqrt{x^2 + y^2 + z^2}, \arctan(\frac{y}{x}), \arctan(\frac{\sqrt{x^2+y^2}}{z}))$$

SURFACES IN SPHERICAL COORDINATES. $g(\rho, \theta, \phi) = 0$ defines a surface in spherical coordinates.

$\rho = \phi$, an **apple**.
$\rho = 1$, a **sphere**.
$\phi = \pi/4$, a **cone**.

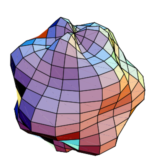






SPHERICAL GRAPHS. A surface $\rho = f(\theta, \phi)$ is called a **spherical graph**.

$\rho(\theta, \phi) = 1 + \sin(5\theta)\sin(7\phi)/2$ **bumpy sphere**.



LATTITUDE AND LONGITUDE. A point on the earth is described by the coordinates **latitude** and **longitude** (ρ, lat, lon) is related to spherical coordinates by $(\rho, \theta, \phi) = (\rho, -lon, 90 - lat)$. For example, $(\rho, lat, lon) = (6365, 42.365, -71.1)$ is the location of Harvard.