Completion of the square.

Reminder: The quadratic equation $x^2+bx+c=0$ is solved by adding $(b/2)^2-c$ on both sides:

$$\begin{aligned} x^2+bx+c &= 0 \\ x^2+bx+(b/2)^2 &= (b/2)^2-c \\ (x+b/2)^2 &= (b/2)^2-c \\ x+b/2 &= \sqrt{(b/2)^2-c} \\ x &= \pm\sqrt{(b/2)^2-c}-b/2\ . \end{aligned}$$

This method is called **completion of the square**.

One can use this to determine the center and radius of circle or spheres.

Example: $x^2+5x+y^2-2y=-1$ is equivalent to $(x+5/2)^2-25/4+(y-1)^2-1=-1$ or $(x-5/2)^2+(y-1)^2=(5/2)^2$. Therefore, the equation is a circle with center $(5/2,1)$ and radius $5/2$.

Now it is your turn:

PROBLEM. Find the center and radius of the sphere $x^2+6x+y^2-4y+z^2-2z=4$.

1) Completion of square for x:

2) Completion of square for y:

3) Completion of square for z:

Result:

The equation is therefore a sphere of radius:

centered at the point: P=