**KNOTS** **O. Knill, Maths21a**

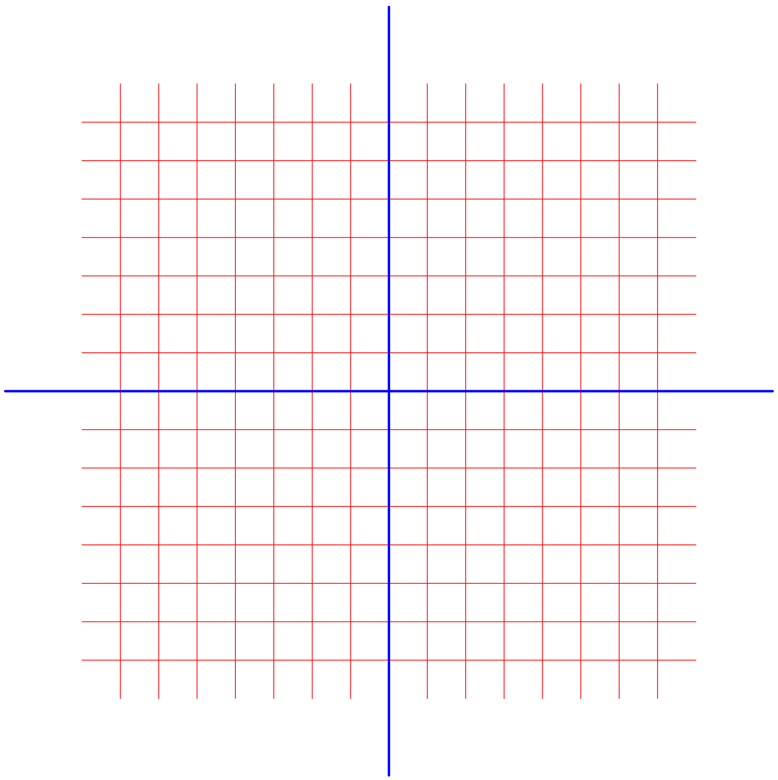
The **threefoil knot** is the space curve

$$\vec{r}(t) = ((2 + \cos(3t/2))\cos(t), (2 + \cos(3t/2))\sin(t), \sin(3t/2))$$

1) Find an interval $[a, b]$ on which $\vec{r}(t)$ parameterizes a closed curve in space. A closed curve in space is called a **knot**. These objects are not only interesting in mathematics or physics. For example, DNA of bacteria and some proteins form knots (see the back of the page).

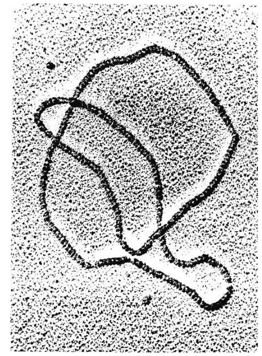
2) Verify that the projection $\vec{r}(t) = \langle x(t), y(t) \rangle$ of this curve onto the $xy$-plane is in polar coordinates by $\vec{r}(t) = 2 + \cos(3t/2), \theta(t) = t$.

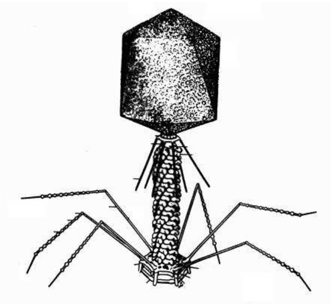
3) Sketch this curve.

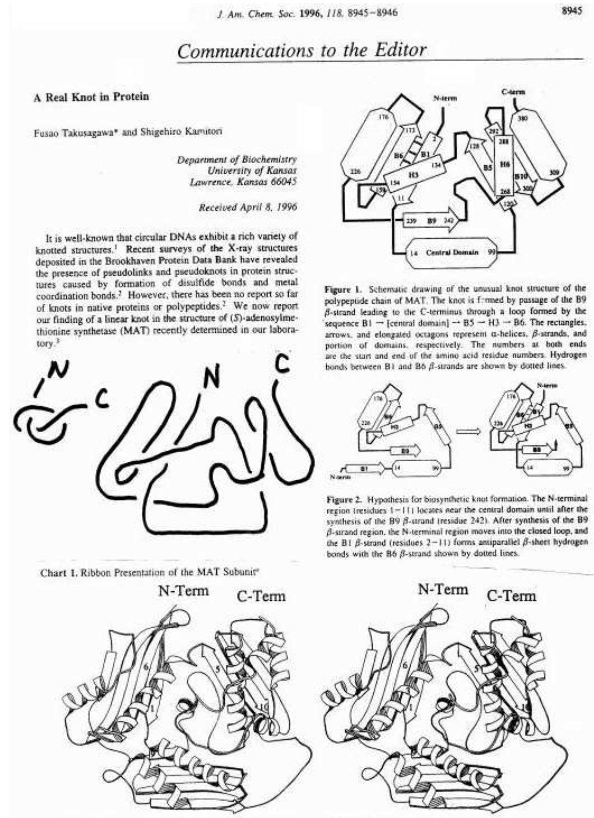


Indicate at the crossings, which part of the curve is above the other.

4) Calculate the velocity vector $\vec{r}'(t)$ of $\vec{r}(t)$ at $t = 0$. If this vector is $\vec{v} = \langle a, b, c \rangle$, then $(a, b)$ is its projection on the $xy$ plane. Draw this vector $\langle a, b \rangle$ in the above $xy$-projection of the knot.







J. Am. Chem. Soc. 1996, 118, 8945−8946 8945

*Communications to the Editor*

**A Real Knot in Protein**

Fusao Takusagawa* and Shigehiro Kamitori

*Department of Biochemistry*
*University of Kansas*
*Lawrence, Kansas 66045*

*Received April 8, 1996*

It is well-known that circular DNAs exhibit a rich variety of knotted structures.[1] Recent surveys of the X-ray structures deposited in the Brookhaven Protein Data Bank have revealed the presence of pseudolinks and pseudoknots in protein structures caused by formation of disulfide bonds and metal coordination bonds.[2] However, there has been no report so far of knots in native proteins or polypeptides.[2] We now report our finding of a linear knot in the structure of (*S*)-adenosylmethionine synthetase (MAT) recently determined in our laboratory.[3]



**Figure 1.** Schematic drawing of the unusual knot structure of the polypeptide chain of MAT. The knot is formed by passage of the B9 $\beta$-strand leading to the C-terminus through a loop formed by the sequence B1 → [central domain] → B5 → H3 → B6. The rectangles, arrows, and elongated octagons represent $\alpha$-helices, $\beta$-strands, and portion of domains, respectively. The numbers at both ends are the start and end of the amino acid residue numbers. Hydrogen bonds between B1 and B6 $\beta$-strands are shown by dotted lines.

**Figure 2.** Hypothesis for biosynthetic knot formation. The N-terminal region (residues 1−11) locates near the central domain until after the synthesis of the B9 $\beta$-strand (residue 242). After synthesis of the B9 $\beta$-strand region, the N-terminal region moves into the closed loop, and the B1 $\beta$-strand (residues 2−11) forms antiparallel $\beta$-sheet hydrogen bonds with the B6 $\beta$-strand shown by dotted lines.

Chart 1. Ribbon Presentation of the MAT Subunit[a]



For more information on DNA knots, see the MSRI talk by DeWitt Sumners:
http://www.msri.org/publications/ln/msri/2000/molbio/sumners/1/