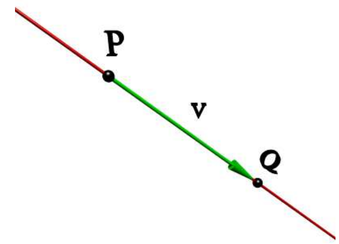
LINES. A point P and a vector $\vec{v}$ define a line L. It is the set of points



$$L = \{P + t\vec{v}, \text{ where } t \text{ is a real number } \}$$

The line contains the point P and points into the direction $\vec{v}$.

EXAMPLE. $L = \{(x, y, z) = (1, 1, 2) + t(2, 4, 6)\ \}$.

This description is called the **parametric equation** for the line.

EQUATIONS OF LINE. We can write $(x, y, z) = (1, 1, 2) + t(2, 4, 6)$ so that $x = 1 + 2t, y = 1 + 4t, z = 2 + 6t$. If we solve the first equation for t and plug it into the other equations, we get $y = 1 + (2x - 2), z = 2 + 3(2x - 2)$. We can therefore describe the line also as

$$L = \{(x, y, z) \mid y = 2x - 1, z = 6x - 4\ \}$$

SYMMETRIC EQUATION. The line $\vec{r} = P + t\vec{v}$ with $P = (x_0, y_0, z_0)$ and $\vec{v} = (a, b, c)$ satisfies the **symmetric equations**

$$\frac{x-x_0}{a} = \frac{y-y_0}{b} = \frac{z-z_0}{c}$$

Proof. Every of the three expression is equal to t.

PROBLEM. Find the equations for the line through the points $P = (0, 1, 1)$ and $Q = (2, 3, 4)$.

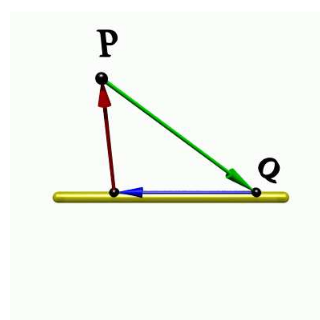
SOLUTION. The parametric equations are $(x, y, z) = (0, 1, 1) + t(2, 2, 3)$ or $x = 2t, y = 1 + 2t, z = 1 + 3t$. Solving each equation for t gives the symmetric equations $x/2 = (y - 1)/2 = (z - 1)/3$.

DISTANCE POINT-LINE (3D). If P is a point in space and L is the line $\vec{r}(t) = Q + t\vec{u}$, then

$$d(P, L) = |(P - Q) \times \vec{u}|/|\vec{u}|$$

is the distance between P and the line L.

This formula is verified by writing $(P - Q) \times \vec{u} = |P - Q||\vec{u}| \sin(\theta)$.



EXAMPLE. $P = (1, 3, 1)$ is a point in space and L is the line $\vec{r}(t) = (1, 1, 1) + t(1, 0, 1)$. Then

$$d(P, L) = |(0, 2, 0) \times (1, 0, 1)|/\sqrt{2} = |(2, 0, -2)|/\sqrt{2} = \sqrt{8}/\sqrt{2} = 2$$

is the distance between P and L.