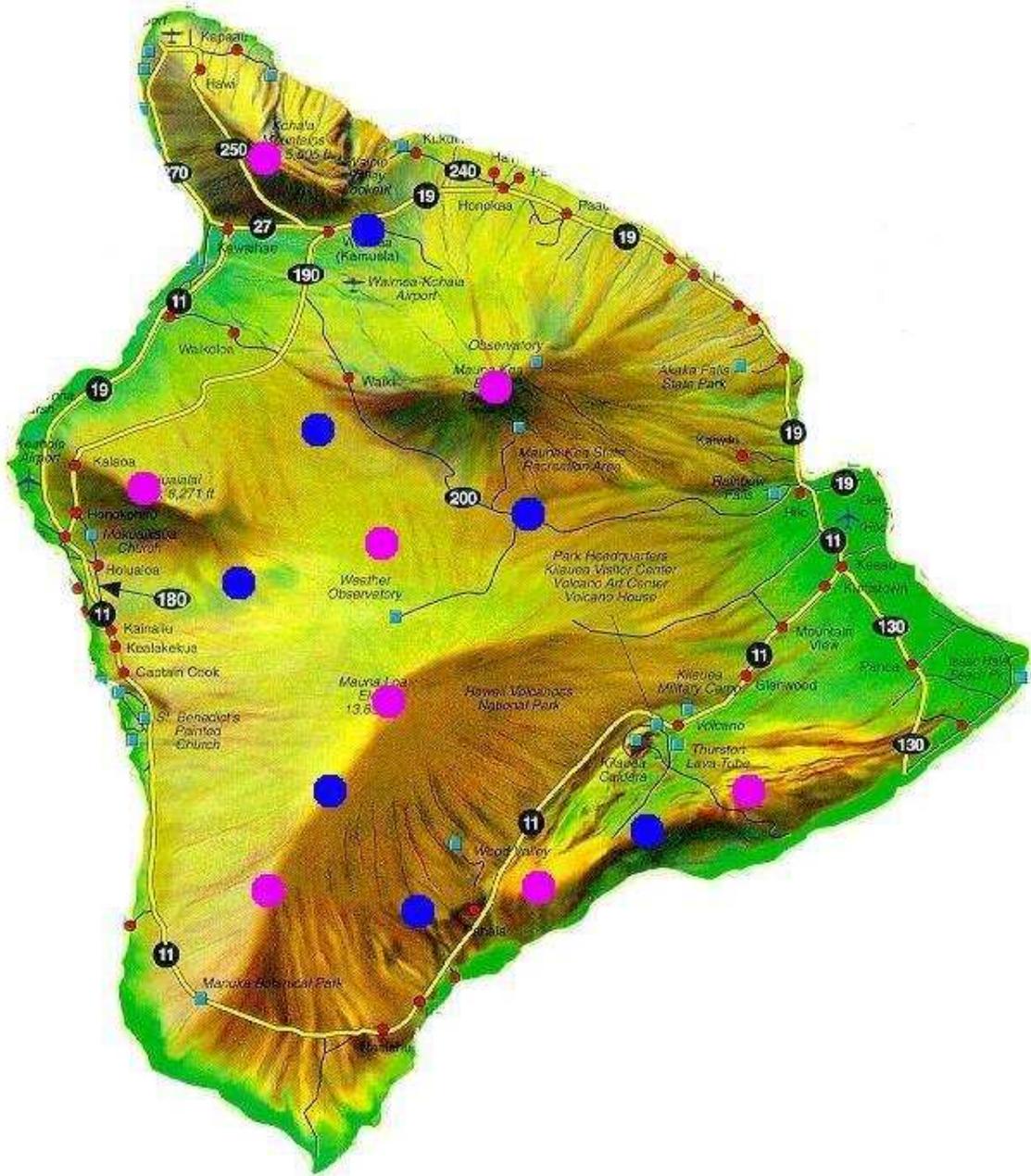


Find all the critical points of  $f(x, y) = x^5y + xy^5 + xy$  and classify them.

- 1) Find the gradient  $\nabla f(x, y)$ .
- 2) Find the critical points  $\nabla f(x, y) = (0, 0)$ .
- 3) Find the discriminant  $D$  at each critical point.
- 4) If  $D > 0$ , find the sign of  $H_{11}$ .
- 5) Classify the critical points.

You are on a circular island surrounded by sea. You count  $A$  mountain peaks,  $B$  sinks and  $C$  mountain passes. You observe that  $A + B - C = 1$ . Can you see why this is true for any island?



The example shows the Hawaii main island. On a rough scale, we have  $A = 7$  mountain peaks,  $B = 1$  sinks and  $C = 7$  mountain passes.

- 1) Explore the rule with simple examples.
- 2) Look what happens if you deform a given island to an island, with only one mountain and no sink and no mountain pass.
- 3) Can you see what happens during the deformation?