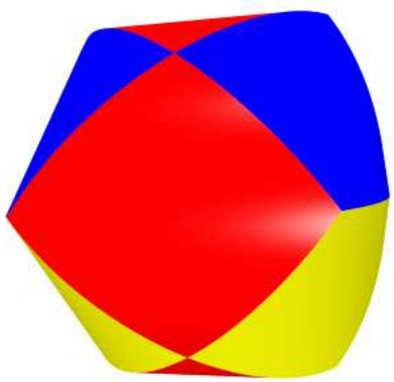
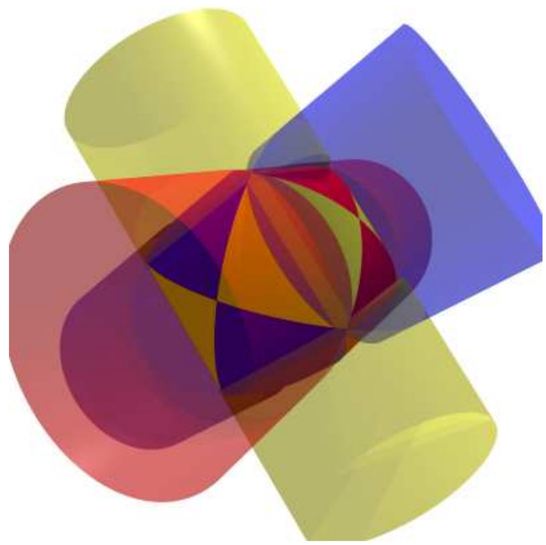
THREE CYLINDERS **Maths21a**

Problem: Find the body obtained by intersecting the three solid cylinders $x^2+z^2 \leq 1$, $y^2+z^2 \leq 1$ and $x^2+z^2 \leq 1$.

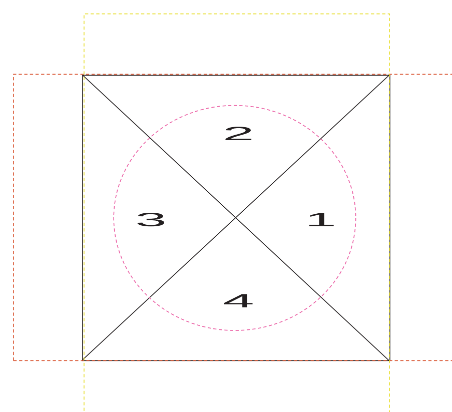




The problem gets easier with the advise:

- make a good picture of the situation
- use symmetry to simplify the problem.

We first look at the intersection R of the cylinders around the x-axes and the y-axis from above. You see 4 equal parts. When considering the part above and below the xy-plane separately, one can find an integral which is 1/8 of the answer.



The third cylinder $x^2+y^2 \leq 1$ appears as a circle in this picture.
To compute the volume, we work only in one quarter of the body and where $z > 0$. The radius goes from 0 to a, the angle from $-\pi/4$ to $\pi/4$. The equation of the roof of the body is $z = \sqrt{1-x^2}$, because in region 1, only the cylinder $x^2+z^2 = 1$ matters. In cylindrical coordinates, not forgetting the integration factor r, and $x = r\cos(\theta)$ and to multiply everything by 8.

We use cylindrical coordinates

$$8\int_{-\pi/4}^{\pi/4}\int_0^1 \sqrt{1-r^2\cos^2(t)}r\ drdt$$

The inner integral over r is $(8/3)(1-|\sin(t)|^3)/\cos^2(t)$ and then we get

$$(16/3)\int_0^{\pi/4}(1-\sin(t)^3)/\cos^2(t)\ dt = (16/3)[\tan(t)-\cos(t)-1/\cos(t)]|_0^{\pi/4} = 16-8\sqrt{2}$$