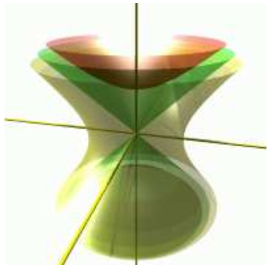
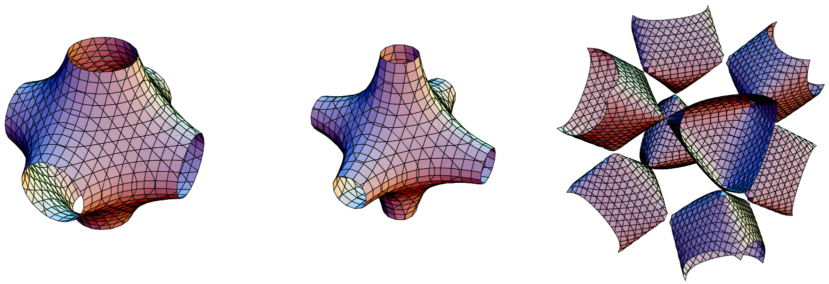
IMPLICIT SURFACES. Given a function $g(x, y, z)$ of three variables, we can look at the surface $g(x, y, z) = c$. It is called a **level surface** of g. We have seen planes or quadrics of this form already.

EXAMPLE. Let $f(x, y, z) = x^2 + y^2 - z^2$. $f(x, y, z) = 0, f(x, y, z) = 1, f(x, y, z) = -1$. The set $x^2+y^2-z^2 = 0$ is a **cone** rotational symmetric around the z-axis. The set $x^2 + y^2 - z^2 = 1$ is a **one-sheeted hyperboloid**, the set $x^2 + y^2 - z^2 = -1$ is a **two-sheeted hyperboloid**. (To see that it is two-sheeted note that the intersection with $z = c$ is empty for $-1 \leq z \leq 1$.)



FLASHBACK TO LEVEL CURVES. Implicit surfaces $g(x, y, z) = c$ are the analogue of implicit curves $f(x, y) = c$ in the two dimensional plane. Unlike for functions in two dimensions, where we could also draw the graph of f as a surface, we can no more draw the graph of a function $g(x, y, z)$ of three dimensions. But drawing several level surfaces gives a good picture, how the function g behaves.

MORE EXAMPLES.
1) $g(x, y, z) = ax + by + cz = d$ plane normal to the vector (a, b, c).
2) $g(x, y, z) = z - f(x, y)$ graph of f
3) $g(x, y, z) = x^2 + y^2 + z^2 = 1$ sphere
4) $g(x, y, z) = x^2 + y^2 - z^2 = 1$ hyperboloid
5) $g(x, y, z) = x^4 + y^4 + z^4 - y^2z^2 - z^2x^2 - x^2y^2 - x^2 - y^2 - z^2 + 1 = 0$ tetraoid, a Kummer surface 6) $g(x, y, z) = x^2y^2 + y^2z^2 + z^2x^2 = 3a$ an other Kummer surface



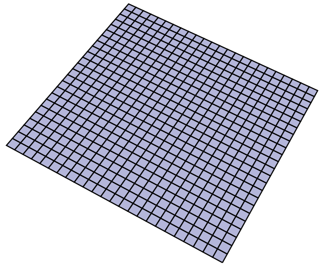
PLOTTING WITH MATHEMATICA. Level curves of functions of two variables can be displayed using the "ParametricPlot3D" command. Unlike with "ParametricPlot" one has to load a library first to execute the command. The examples above are obtained using this command.

SWITCHING FROM IMPLICIT TO PARAMETRIC. In general, it is not easy to switch for and back from implicit to parametric. It would need quite a bit of time to come up with a parameterization of the surface $g(x, y, z) = x^3 + y^4 \sin(x)z + z^3 = 1$.
This is by the way already a difficult problem for level curves for which we will look at parameterizations later. There are some surfaces, where we know both the parametric and implicit form.

PLANE.

Parametric: $r(u, v) = (P_1+uu_1+vv_1, P_2+uu_2+vv_2, P_3+uu_3+vv_3)$
Implicit: $ax + by + cz = d$.

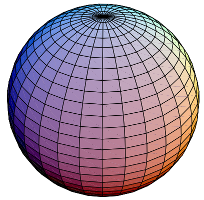
Parametric to Implicit: $(a, b, c) = (u_1, u_2, u_3) \times (v_1, v_2, v_3)$.
Implicit to Parametric: Find three points P, Q, R on the surface and form $\vec{u} = \vec{PQ}, \vec{v} = \vec{PR}$.



SPHERE.

Parametric: $r(u, v) = (\rho \cos(u) \sin(v), \rho \sin(u) \sin(v), \rho \cos(v))$
Implicit: $x^2 + y^2 + z^2 = \rho^2$.

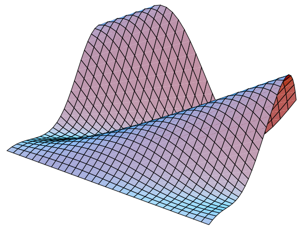
Parametric to Implicit: read of radius.
Implicit to Parametric: know it



GRAPH.

Parametric: $r(u, v) = (u, v, f(u, v))$
Implicit: $z - f(x, y) = 0$.

Parametric to Implicit: think about $z = f(x, y)$
Implicit to Parametric: use x and y as the parameterizations.



SURFACE OF REVOLUTION.

Parametric: $r(u, v) = (g(v) \cos(u), g(v) \sin(u), v)$
Implicit: $\sqrt{x^2 + y^2} = r = g(z)$ can be written as $x^2 + y^2 = g(z)^2$.

Parametric to Implicit: read off the function $g(z)$ the distance to the z-axis.
Implicit to Parametric: use the function g.