Final Homework Assignment

Please note that these problems are due at the beginning of class on Wenesday December 12. If you are in the Tuesday-Thursday section, the problems are due in Alexey Gorshkov's mail box by noon on Wednesday December 12. (Alexey's mail box is near Science Center 308.)

In Questions 1, 2 and 3 find explicit formulas for the functions that solve the initial value problems.

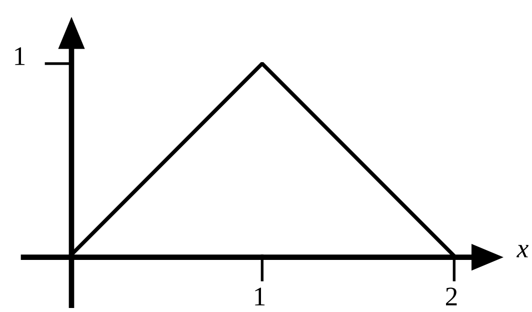
1. $y'' + y' - 2y = 0.$ $y(0) = 1,\ y'(0) = 1.$

2. $6y'' - 5y' + y = 0.$ $y(0) = 4,\ y'(0) = 0.$

3. $y'' + 8y' - 9y = 0.$ $y(1) = 1,\ y'(1) = 0.$

The point of Questions 4, 5 and 6 is to calculate the Fourier Series of a periodic function, and then verify that the function defined by the Fourier series formula really does match up with the periodic function.

In these questions, you will be dealing with a periodic function, $f(x)$. The graph of **one** repetition of the function is given below.



4. Calculate the coefficient of $\sin\left(\frac{n\pi x}{2}\right)$ (where n is a positive integer) in the Fourier Series of f. You should show full details of your calculation and only use technology to check your answer (if you use it at all).

5. Calculate the coefficient of $\cos\left(\frac{n\pi x}{2}\right)$ (where $n \geq 0$ is an integer) in the Fourier Series of f. You should show full details of your calculation and only use technology to check your answer (if you use it at all).

6. Three functions are defined below. The a_n refer to the numbers that you calculated in Question 5 and the b_n to the numbers that you calculated in Question 4. Sketch a graph of each of the functions over the interval [0, 4].

$$g_3(x) = a_0 + \sum_{n=1}^{3}\left[a_n \cdot \cos\left(\frac{n\pi x}{2}\right) + b_n \cdot \sin\left(\frac{n\pi x}{2}\right)\right]$$

$$g_{10}(x) = a_0 + \sum_{n=1}^{10}\left[a_n \cdot \cos\left(\frac{n\pi x}{2}\right) + b_n \cdot \sin\left(\frac{n\pi x}{2}\right)\right]$$

The point of Questions 7, 8, 9 and 10 is to find a non-trivial solution $u(x, t)$ to the problem:

$$5\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t} \qquad \text{for } 0 \le x \le \pi \text{ and } t > 0.$$

$u(0, t) = 0$ and $u(\pi, t) = 0$ for $t > 0$.

$u(x, 0) = \sin(2x)$ for $0 \le x \le \pi$.

NOTE: You should supply details of any intermediate calculations that you do, and carefully record any assumptions that you make about the signs of constants.

7. Use the technique of separation of variables:

$$u(x, t) = X(x) \cdot T(t)$$

to find a solution to the partial differential equation given above.

8. Apply the boundary conditions:

$$u(0, t) = 0 \text{ and } u(\pi, t) = 0 \text{ for } t > 0.$$

to simplify the equations from Question 7.

9. Use Fourier series and the initial condition:

$$u(x, 0) = \sin(2x) \text{ for } 0 \le x \le \pi.$$

to identify any unspecified constants in your solution $u(x, t)$.

10. Using the results from Questions 7, 8 and 9, write down the formula for the solution to the partial differential equation (that also satisfies the initial and boundary conditions given).