Practice Problems: Test #1

Please note: *I have chosen these problems because I think that they are representative of many of the mathematical concepts that we have studied. There is no guarantee that the problems that appear on any test will resemble these problems in any way whatsoever.*

Remember: *On exams, you have to supply evidence for your conclusions, and explain why your answers are appropriate.*

1. The shopping lists of four graduate students are listed below. These shopping lists give the quantities of each item that the graduate students want to buy.

Item	Mike	Tim	Chris	Ron
Potatoes	3	1	0	1
Tooth paste	1	0	1	0
Beverages	10	0	4	0
Meat	0	1	1	0
Lettuce	2	3	0	2

The prices (in dollars) of each unit of each item at three area supermarkets are given below.

Item	Super-Value	New World	Woolworths
Potatoes	0.16	0.14	0.16
Tooth paste	1.50	1.44	1.50
Beverages	0.50	0.70	0.50
Meat	3.34	3.40	4.00
Lettuce	0.30	0.26	0.20

When you are a graduate student, you have to make your dollars go as far as possible. Where should each student shop to get the items they need at the smallest possible cost?

2. Ann Arbor is a notoriously liberal town in southeastern Michigan. The mayoral elections are usually contested between the Democratic, D, and Republican, R, parties. Occasionally, an independent candidate runs, but this is very rare. Political scientists who are interested in patterns in voter behavior notice the following pattern in at each election:

(I) Of those who voted D in the last election, 75% will vote D in the next election, and 25% will vote R in the next election.

(II) Of those who voted R in the last election, 80% will do so again in the next election and 20% will vote D in the next election.

In 1998, 60% of voters voted D, and 40% voted R.

(a) Who should win this year's election?

(b) Who should win the election in 2002?

3. An n by n magic square is an array of non-negative integers arranged so that each column and row adds up to the same number S. For example, the matrix given below is a 3 by 3 magic square with $S = 6$:

$$\begin{bmatrix} 1 & 0 & 5 \\ 5 & 1 & 0 \\ 0 & 5 & 1 \end{bmatrix}.$$

Suppose that A and B are n by n magic squares with sums S_1 and S_2 respectively.

(a) Show that $A + B$ is a magic square with sum $S_1 + S_2$.

(b) Show that AB is a magic square with sum S_1S_2.

4. Let A and B be the matrices given below.

$$A = \begin{bmatrix} 1 & 3 & 8 \\ 2 & 4 & 11 \\ 1 & 2 & 5 \end{bmatrix} \qquad B = \begin{bmatrix} -3 & 5 \\ 1 & 5 \\ 3 & 4 \end{bmatrix}.$$

(a) Explain how you could possibly calculate the matrix $A^{-1}B$ without figuring out A^{-1}.

(b) Use your method to calculate the matrix $A^{-1}B$.

5. Suppose that v_1, v_2 and v_3 are three dimensional vectors that do not all lie in the same plane. Suppose that none of these vectors are the zero vector, and suppose that:

$$a_1v_1 + a_2v_2 + a_3v_3 = b_1v_1 + b_2v_2 + b_3v_3\ ,$$

where the a's and b's are real numbers. Show that $a_j = b_j$ for $j = 1, 2, 3$.

6. Find a vector that is parallel to the intersection of the planes:

$$x - y + z = 5$$

$$2x + y - 3z = 4.$$

7. Use matrices and row reduction to prove that any set of two dimensional vectors has to be linearly dependent if the set has three or more vectors in it.

8. Determine values of λ and μ so that the points (-1, 3, 2), (-4, 2, -2) and (5, λ, μ) lie on a straight line.

9. Find a value of λ that will make the three vectors:

$$\mathbf{a} = \mathbf{I} + \mathbf{j} + \mathbf{k}$$

$$\mathbf{b} = 2\mathbf{I} - 4\mathbf{k}$$

$$\mathbf{c} = \mathbf{I} + \lambda\mathbf{j} + 3\mathbf{k}$$

coplanar.

10. In each of the following, either find the general solution of the system of equations or show that the system has no solution.

(a) $3x - 2y + z = 6$
$x + 10y - z = 2$
$-3x - 2y + z = 0$

(b) $2x_1 - 3x_2 + x_4 - x_6 = 0$
$3x_1 - 2x_3 + x_5 = 1$
$x_2 - x_4 + 6x_6 = 3$

(c) $4x - y + z = -1$
$-3x + y - 5z = 0$
$-5x - 14z = 10$

(d) $4x - 3y + 4z = 1$
$x + y - 5z = 0$
$-2x + y + 7z = 4$

11. For each of the following matrices, either find the inverse of the matrix or show that the matrix does not have an inverse.

(a) $\begin{bmatrix} 6 & 2 \\ 3 & 3 \end{bmatrix}$

(b) $\begin{bmatrix} -3 & 4 & 1 \\ 1 & 2 & 0 \\ 1 & 1 & 3 \end{bmatrix}$

(c) $\begin{bmatrix} -2 & 1 & 1 \\ 0 & 1 & 1 \\ -3 & 0 & 6 \end{bmatrix}$

12. Budget Rent-A-Car in Wichita, Kansas, has a fleet of about 450 cars, at three locations. A car rented at one location may be returned to any of the three locations. The various fractions of cars returned to each location are shown in the table below. Suppose that on Monday, there are 304 cars at the airport (or rented there), 48 cars at the east side office and 98 cars at the west side office. What will be the approximate distribution of cars on Wednesday?

Cars rented from airport	Cars rented from east side office	Cars rented from west side office	
0.97	0.05	0.10	Cars returned to airport
0.00	0.90	0.05	Cars returned to east side office
0.03	0.05	0.85	Cars returned to west side office

13. If a square matrix, *A*, has an inverse, then the matrix is sometimes called non-singular.

(a) Show that the matrix given below is non-singular.

$$A = \begin{bmatrix} 4 & -2 & 5 \\ 1 & 1 & -2 \\ 6 & 1 & -1 \end{bmatrix}$$

(b) Use the inverse of *A* to find a solution for each system of linear equations that follows the pattern:

$$4x_1 - 2x_2 + 5x_3 = a$$

$$x_1 + x_2 - 2x_3 = b$$

$$6x_1 + x_2 - x_3 = c,$$

where *a*, *b* and *c* are real numbers.

(c) Find the solutions of the system of linear equations that correspond to the following choices of *a*, *b* and *c*.

	a	b	c
(I)	0	0	0
(II)	1	0	0
(III)	2	-7	1

(d) Suppose that you are given particular values of *a*, *b* and *c*. How many solutions does the system of linear equations have? How do you know?

14. Decide whether the following statements are true or false. Remember that to be true, the statement must be true for every single matrix described. (A square matrix is called *diagonal* if the entries a_{ij} are zero whenever $i \neq j$.)

(a) If two *n* by *n* matrices have the same reduced row echelon form (RREF) then either both are invertible or both lack inverses.

(b) Diagonal matrices are invertible.

(c) Let $A = [a_1\ a_2\ ...\ a_m]$ be a matrix. Suppose that $\{a_1, ..., a_m, b\}$ is a linearly independent set. Then the system of linear equations $Ax = b$ is consistent.

15. **(a)** Suppose that you know that A is a matrix that has an inverse. Is it true that A^k has an inverse for every single positive integer value k? How do you know?

(b) For the matrix B shown below, calculate B^2 and B^3.

$$B = \begin{bmatrix} -22 & -3 & -25 \\ 40 & 6 & 44 \\ 16 & 3 & 16 \end{bmatrix}$$

(c) Decide, based on the results of part (b), whether or not the matrix B has an inverse.

16. Olympic athletes, in addition to their rigorous training schedules, often have to follow very careful diets as well. Olympic teams will often include a dietician whose job it is to know the nutritional content of foods and to design diets that will help the athletes to reach peak performance. In this problem you'll apply what you have been learning about linear algebra to solve the kinds of problems that dieticians sometimes have to work with.

One serving (28g) of oat bran supplies 110 calories, 3g of protein, 21g of carbohydrate and 3g of fat. One serving of corn flakes supplies 110 calories, 2g of protein, 25g of carbohydrate and 0.4g of fat.

(a) Set up a matrix B and a vector u such that Bu gives the amounts of calories, protein, carbohydrate and fat contained in a mixture of three servings of oat bran and two servings of corn flakes.

(b) Is it possible for a mixture of the two cereals to supply 110 calories, 2.25g of protein, 24g of carbohydrate and 1g of fat? If so, what is the mixture?

17. In data analysis, an *interpolating function* is a function that passes through each of the points in your data set.

(a) Find a quadratic function that passes through the three points (1, 12), (2, 15) and (3, 16).

(b) Is it true that, given three points, you can find a quadratic function that passes through the three points?

(c) Suppose you have three points with the following property:

(I) No two points have the same x coordinate.

Is it true that you can always find a quadratic function that goes through the three points? Justify your answer.

18. In each of the following cases, determine all the values of t that will make the given matrix singular (that is, the matrix does not have an inverse).

(a) $\begin{bmatrix} 8 & -3 & 1 \\ 3 & t & -2 \\ 2 & -5 & 5 \end{bmatrix}$

(b) $\begin{bmatrix} t & 1 & 0 \\ 0 & t & 1 \\ 15 & 17 & t+1 \end{bmatrix}$

(c) $\begin{bmatrix} t-6 & 1 & 5 \\ 0 & t+1 & -1 \\ -9 & 1 & t+8 \end{bmatrix}$

(d) $\begin{bmatrix} 3 & t & 1 \\ 3 & 1 & t \\ t+5 & 2 & 2 \end{bmatrix}$

19. Find the rank of the following matrices:

(a) $\begin{bmatrix} -3 & 1 & 5 \\ 2 & 7 & 1 \\ 1 & -3 & 2 \end{bmatrix}$

(b) $\begin{bmatrix} 8 & -3 & 1 & 2 \\ -5 & 1 & 2 & 7 \\ 1 & -3 & 8 & 25 \end{bmatrix}$

(c) $\begin{bmatrix} -7 & 9 & 4 & 5 & 3 \\ 5 & 1 & 0 & -2 & 1 \\ 3 & 11 & 4 & 1 & 5 \\ -5 & -1 & 0 & 2 & -1 \end{bmatrix}$

20. Let: $\alpha_1 = (5, 1, -3)$ $\alpha_2 = (7, -2, 4)$ $\alpha_3 = (1, 0, 1)$

$\beta_1 = (4, -3, 1)$ $\beta_2 = (7, -5, 2)$ $\beta_3 = (1, 1, 1)$

$S = \{\alpha_1, \alpha_2, \alpha_3\}$ $T = \{\beta_1, \beta_2, \beta_3\}$

(a) Find the vector γ whose coordinates with respect to S are (-5, 3, 1).

(b) Find the coordinates of $\rho = (2, -4, 15)$ with respect to T.

(c) Find the coordinates of $\rho = (2, -4, 15)$ with respect to S.

21. Find non-zero integers x, y and z so that the inverse of the matrix:

$$P = \begin{bmatrix} 3 & -1 & 2 \\ 1 & 1 & 5 \\ x & y & z \end{bmatrix}$$

will contain only integers

22. In each of the following cases, decide whether the collection of functions is a subspace of the vector space of real-valued functions of a single variable.

(a) All functions $f(x)$ such that: $\left|\int_0^1 f(x)dx\right| < \infty$.

(b) All solutions of the differential equation: $y'' - (\sin(x))y' + e^x y = 0$.

(c) All functions $f(x)$ with the property that: $f(x) \geq f(y)$ if $x \geq y$.

(d) All functions $f(t)$ that satisfy: $\int_0^1 f(t) \cdot \sin(t)dt = 0$.

(e) All functions that satisfy $f(x) \geq 0$ for all real numbers x.

23. Let α_1, α_2, and α_3 be elements of a vector space V. Let:

$$\beta_1 = 2\alpha_1 - \alpha_2 + 2\alpha_3$$
$$\beta_2 = -2\alpha_1 + \alpha_2 - 5\alpha_3$$

(a) Express $\gamma = 3\beta_1 + 7\beta_2$ as a linear combination of the α's.

(b) Show that each linear combination $s_1\beta_1 + s_2\beta_2$ can be expresses as a linear combination of the α's.

(c) Find a linear combination of the α's that is not in Span$\{\beta_1, \beta_2\}$.

24. Decide whether each of the following statements is true or false.

(a) The zero vector cannot be contained in a basis.

(b) If a set, S, of vectors is a subset of a linearly independent set of vectors, then S is a linearly independent set of vectors.

(c) If a set, T, of vectors is linearly dependent, then any set of vectors that contains T is linearly dependent.

(d) Two linearly independent vectors may be parallel.

(e) Suppose that three vectors are all members of a 2 dimensional subspace of $\boldsymbol{R}^2$. If you choose any two of the three, then you will have a basis for the subspace.

25. The coefficient matrix of a 4 by 5 homogeneous system (i.e., $A\mathbf{x} = \mathbf{0}$) of linear equations has the reduced row echelon form:

$$\begin{bmatrix} 1 & -3 & 0 & -5 & 2 \\ 0 & 0 & 1 & 7 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

(a) Find the solutions to the homogeneous system of equations. What is the dimension of the solution space?

(b) Determine whether or not η = (3, 2, 6, -1, -1) is also a solution of the homogeneous system.

(c) A non-homogeneous system that has the same coefficient matrix as the homogeneous system has the solution ρ_0 = (4, -1, 14, 6, -5). Is the vector:

$$\rho = (-3, 3, 15, -2, 1)$$

also a solution of the system?

26. *A* is a 4 by 5 matrix that has rank 3. Let α_1, α_2, α_3, α_4 be vectors that represent the rows of *A*, and vectors ρ_1, ρ_2, and ρ_3 be vectors that satisfy:

$$\begin{aligned} \alpha_1 &= -7\rho_1 + 2\rho_2 - 5\rho_3 \\ \alpha_2 &= 4\rho_1 + 0\rho_2 + \rho_3 \\ \alpha_3 &= 9\rho_1 + \rho_2 - 2\rho_3 \\ \alpha_4 &= -2\rho_1 + \rho_2 + 3\rho_3 \end{aligned}$$

(a) The *row space* of a matrix is the span of the vectors that represent the rows of the matrix. Show that $\{\rho_1, \rho_2, \rho_3\}$ is a basis for the row space of *A*.

(b) Find three vectors that span the image of *A*.

(c) Determine whether or not the linear system $Ax = \beta$ has a solution if:

$$\beta = \begin{bmatrix} 0 \\ 3 \\ 13 \\ -4 \end{bmatrix}.$$

27. Often in economics and biology, researchers model economic and biological phenomena with differential equations. Because of the complexity of economics and biology, systems of differential equations are often required (i.e. the phenomena cannot be adequately described using a single differential equation).

In this problem, we will consider functions $x(t)$ and $y(t)$ that are defined by the differential equations:

$$2\frac{dx}{dt} + 5\frac{dy}{dt} = t$$

$$\frac{dx}{dt} + 3\frac{dy}{dt} = 7\cos(t).$$

(a) Re-write the system of differential equations as an equation involving matrices and vectors.

(b) Use a matrix inverse to find equations for $\frac{dx}{dt}$ and $\frac{dy}{dt}$ that involve only constants, *t*, and cos(*t*).

(c) Find formulas for $x(t)$ and $y(t)$ if: $x(0) = 0$ and $y(0) = 0$.

28. Let $L:\boldsymbol{R}^3 \rightarrow \boldsymbol{P}_2$ be the function defined by:

$$L\left(\begin{bmatrix} a \\ b \\ c \end{bmatrix}\right) = (b+c)t^2 + (-a-2b-c)t + (a+b)\ .$$

(a) Is *L* an example of a linear transformation? Explain.

(b) Is *L* invertible? If so, find a formula for $L^{-1}:\boldsymbol{P}_2 \rightarrow \boldsymbol{R}^3$.

(c) What is the dimension of the image of *L*?

(d) Find a basis for the kernel of *L*.

Brief Answers (Too brief to score well on an exam.)

1. Mike - Woolworths
Chris - Super-Value
Tim - New World
Ron - New World

2.(a) The Democrats should win.
2.(b) The Republicans should win.

3.(a) Let $C = A + B$. The the (i,j) entry of *C* is: $c_{ij} = a_{ij} + b_{ij}$. Adding along the i^{th} row then gives:

$\sum_{j=1}^{n} c_{ij} = \sum_{j=1}^{n}\left(a_{ij} + b_{ij}\right) = \sum_{j=1}^{n} a_{ij} + \sum_{j=1}^{n} b_{ij} = S_1 + S_2$. The proof that the sum down columns is always $S_1 + S_2$ is similar.

3.(b) Let $C = AB$. Then the (i,j) entry of *C* is: $c_{ij} = \sum_{k=1}^{n} a_{ik}b_{kj}$. Adding across the i^{th} row then gives:

$\sum_{j=1}^{n} c_{ij} = \sum_{j=1}^{n}\left(\sum_{k=1}^{n} a_{ik}b_{kj}\right) = \left(\sum_{k=1}^{n} a_{ik}\right)\left(\sum_{j=1}^{n} b_{kj}\right) = S_1 \cdot S_2$. Adding down columns is similar.

4.(a) This matrix is the solution of the matrix equation $Ax = B$.

4.(b) $A^{-1}B = \begin{bmatrix} 10 & -1 \\ 9 & 10 \\ -5 & -3 \end{bmatrix}$.

5. Subtract one side from the other. The three vectors are linearly independent, so the coefficients of the vector equation must each be zero.

6. Any scalar multiple of <2, 5, 3>.

7. Read the proof of Theorem 8 on pages 62 of your text.

8. $\lambda = 5$ and $\mu = 10$.

9. $\lambda = 5/3$.

10.(a) $x = 1$, $y = 0.5$, $z = 4$.
10.(b) (-4, -4, -38, -11/2, 0, 0) + x_5*(2, 2, 7, 3/2, 1, 0) + x_6*(-2, -1, -9/2, -3/4, 0, 1)
10.(c) No solution.
10.(d) $x = 16/57$, $y = 99/57$, $z = 23/57$.

11.(a) $\begin{bmatrix} 0.25 & -0.166 \\ -0.25 & 0.5 \end{bmatrix}$

11.(b) $\frac{1}{31}\begin{bmatrix} -6 & 11 & 2 \\ 3 & 10 & -1 \\ 1 & -7 & 10 \end{bmatrix}$

11.(c) $\frac{1}{12}\begin{bmatrix} -6 & 6 & 0 \\ 3 & 9 & -2 \\ -3 & 3 & 2 \end{bmatrix}$

12. About 310 cars at the airport, 48 cars at the east side office and 92 cars at the east side office.

13.(a) If you calculate the rref of this matrix, it is the 3 by 3 identity matrix.

13.(b) $(x_1, x_2, x_3) = A^{-1}(a, b, c)$, where $A^{-1} = \begin{bmatrix} 1 & 3 & -1 \\ -11 & -34 & 13 \\ -5 & -16 & 6 \end{bmatrix}$

13.(c)

a	b	c	Solution
0	0	0	(0, 0, 0)
1	0	0	(1, -11, -5)
2	-1	0	(-20, 229, 108)

13.(d) The solutions are unique for every particular choice of (*a*, *b*,*c*). One way to see this: If a matrix is invertible, then it is row equivalent to the *n* by *n* identity matrix. This means that the system of linear equations has exactly the same number of 'basic' equations as it has unknowns. This means that there are no free variables in the system of linear equations. In order for there to be more than one solution to a system of linear equations to have more than one solution, it has to have free variables. Since this system doesn't have any free variables, it can have only one solution.

14.(a) True.
14.(b) False.
14.(c) False.

15.(a) If you have $A^2 = A*A$, then the inverse would be $A^{-1}*A^{-1}$. Let's suppose that A^k has an inverse for some value of A. Call this inverse Q. Then because matrix multiplication is associative, the inverse of A^{k+1} is $A^{-1}*Q$. (You can verify this by straight forward matrix multiplication of $A^{k+1} = A^k*A$ with $A^{-1}*Q$.) The validity of the proposition then follows by the principle of mathematical induction.

15.(b) $B^2 = \begin{bmatrix} -36 & -27 & 18 \\ 64 & 48 & -32 \\ 24 & 18 & -12 \end{bmatrix}$. $B^3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

15.(c) Suppose that B is invertible. If this is really true then every power of B should be invertible as well. But, B^3 is not invertible, so it is not the case that every power of B is invertible. So, the assumption that B is invertible must be wrong.

16.(a) $B = \begin{bmatrix} 110 & 110 \\ 3 & 2 \\ 21 & 25 \\ 3 & 0.4 \end{bmatrix}$. $u = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$

16.(b) The nutrition cannot be provided by a whole number of servings of either cereal. If you are allowed to mix cereals, then about 0.244 of a serving of oat bran and 0.755 of a serving of corn flakes would supply the required nutrition. (These figures were located by guessing and checking.)

17.(a) $p(x) = 7 + 6x - x^2$.
17.(b) No. For example, if two of the points have the same x coordinate and different y coordinates then no function will pass through both of these two points.
17.(c) Suppose that the three points are (x_1, y_1), (x_2, y_2) and (x_3, y_3). To show that you can find a quadratic function $p(x) = ax^2 + bx + c$, you need to show that the system of linear equations:

$$\begin{bmatrix} 1 & x_1 & x_1^2 \\ 1 & x_2 & x_2^2 \\ 1 & x_3 & x_3^2 \end{bmatrix} \begin{bmatrix} c \\ b \\ a \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

is consistent. To do this, it is sufficient to show that the columns of the matrix span R^3. Suppose not, i.e. there is a non-trivial linear combination of the columns of the matrix which equals the zero matrix. Suppose that the coefficients of the columns are h, j, and k.

Case 1 ($k = 0$): Then j cannot be zero. So, $h + jx_1 = 0$ and $h + jx_2 = 0$. But x_1 and x_2 are different so these equations cannot both be satisfied.

Case 2 (k is not zero): Then x_1, x_2 and x_3 must be one of the results given by the quadratic formula:

$$\frac{-j \pm \sqrt{j^2 - 4kh}}{2k}.$$

Since there are at most two possibilities here, and there are three values x_1, x_2 and x_3, then at least two of the x values must be equal. But all of the x values are supposed to be different.

The upshot of these two cases are that the assumption that the columns of the matrix are linearly dependent must be faulty. Therefore, the columns of the matrix must span R^3, and the system must be consistent.

18.(a) $t = 1$.
18.(b) $t = -5, 3, 1$.
18.(c) $t = 0, -3$.
18.(d) $t = 1, -7$.

19.(a) Rank = 3.
19.(b) Rank = 2.
19.(c) Rank = 2.

20.(a) $\gamma = (-3, -11, 28)$.
20.(b) $[\rho]_T = (-186, 109, -17)$.
20.(c) $[\rho]_S = (-2, 1, 5)$.

21. For example, $x = -3$, $y = 2$ and $z = 1$ will work.

22.(a) yes
22.(b) yes
22.(c) no
22.(d) yes
22.(e) no

23.(a) $\gamma = -8\alpha_1 + 4\alpha_2 - 29\alpha_3$.
23.(b) Let s_1 and s_2 be scalars. $s_1\beta + s_2\beta = (2s_1 - 2s_2)\alpha_1 + (-s_1 + s_2)\alpha_2 + (2s_1 - 5s_2)\alpha_3$.
23.(c) $\alpha_1 + \alpha_2 + \alpha_3$.

24.(a) true
24.(b) true
24.(c) true
24.(d) false
24.(e) false

25.(a) (3, 1, 0, 0, 0), (5, 0, -7, 1, 0), (-2, 0, 1, 0, 1).
25.(b) Yes.
25.(c) No.

26.(a) If the ρ's were linearly dependent then the rank of A would be less than 3.
26.(b) $\gamma_1 = (-7, 4, 9, -2)$. $\gamma_2 = (2, 0, 1, 1)$. $\gamma_3 = (-5, 1, -2, 3)$.
26.(c) $Ax = \beta$ is inconsistent since β cannot be expressed as a linear combination of the γ_j's.

27.(a) The equation involving matrices and vectors is:

$$\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \end{bmatrix} = \begin{bmatrix} t \\ 7 \cdot \cos(t) \end{bmatrix}.$$

27.(b) Multiplying both sides of the matrix equation by the inverse matrix gives:

$$\begin{bmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \end{bmatrix} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} t \\ 7 \cdot \cos(t) \end{bmatrix}.$$

Multiplying this out gives that:

$$\frac{dx}{dt} = 3t - 35 \cdot \cos(t)$$

and

$$\frac{dy}{dt} = -t + 14 \cdot \cos(t).$$

27.(c) Integrating the results from Part (b) and applying the conditions $x(0) = 0$ and $y(0) = 0$ gives:

$$x(t) = 1.5*t^2 - 35 \cdot \sin(t)$$

$$y(t) = -0.5*t^2 + 14 \cdot \sin(t).$$

28.(a) Yes as it satisfies both $L(\mathbf{x} + \mathbf{y}) = L(\mathbf{x}) + L(\mathbf{y})$ and $L(k \cdot \mathbf{x}) = k \cdot L(\mathbf{x})$.

28.(b) No.

28.(c) 2.

28.(d) $\left\{ \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix} \right\}$.