True or False

1. T, by Definition on Page 16
2. F; Consider the equation $x + y + z = 0$, repeated four times.
3. F, by Example 3a of Section 1.3
4. T, by Definition 1.3.6
5. T, by Fact 1.3.4.
6. F, by Fact 1.3.1
7. F, by Fact 1.3.4
8. F; As a counter-example, consider the zero matrix.
9. T, by Definition 1.3.6
10. T, by Definition 1.3.5
11. F; The rank is 1
12. F; The product on the left-hand side has two components.

13. T; Let $A = \begin{bmatrix} -3 & 0 \\ -5 & 0 \\ -7 & 0 \end{bmatrix}$, for example.

14. T; We have $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = 2\begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} - \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix}$.

15. T; The last component of the left-hand side is zero for all vectors $\vec{x}$.

16. T; $A = \begin{bmatrix} 3 & 0 \\ 4 & 0 \end{bmatrix}$, for example.

17. T; Find rref

18. T; Find rref

19. F; Consider the 4×3 matrix A that contains all zeroes, except for a 1 in the lower left corner.

20. F; Note that $A\begin{bmatrix} 2 \\ 2 \end{bmatrix} = 2A\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ for all 2×2 matrices A.

21. F; Find rref to see that the rank is always 2.

22. T; Note that $\vec{v} = 1\vec{v} + 0\vec{w}$.

23. F; Let $\vec{u} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \vec{v} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}, \vec{w} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, for example.

24. T; Note that $\vec{0} = 0\vec{v} + 0\vec{w}$

25. F; Let $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, for example. We can apply elementary row operations to A all we want, we will always end up with a matrix that has all zeros in the first column.

26. T; If $\vec{u} = a\vec{v} + b\vec{w}$ and $\vec{v} = c\vec{p} + d\vec{q} + e\vec{r}$, then $\vec{u} = ac\vec{p} + ad\vec{q} + ae\vec{r} + b\vec{w}$.

27. F; The system $x = 2,\ y = 3,\ x + y = 5$ has a unique solution.

28. F; Let $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, for example.

29. F; Let $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}$, for example.

30. T, by Example 13 of Section 1.3 (note that rank(A)≤ 3)

31. T; By Example 3c of Section 1.3, the equation $A\vec{x} = \vec{0}$ has the unique solution $\vec{x} = \vec{0}$. Now note that $A(\vec{v} - \vec{w}) = \vec{0}$, so that $\vec{v} - \vec{w} = \vec{0}$ and $\vec{v} = \vec{w}$.

32. T; Note that rank$(A) = 4$, by Fact 1.3.4.

33. F; Let $\vec{u} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $\vec{w} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, for example.

34. T; We use rref to solve the system $A\vec{x} = \vec{0}$ and find $\vec{x} = \begin{bmatrix} -2t \\ -3t \\ t \end{bmatrix}$, where t is an arbitrary constant.

Letting $t = 1$, we find $[\vec{u}\ \vec{v}\ \vec{w}] \begin{bmatrix} -2 \\ -3 \\ 1 \end{bmatrix} = -2\vec{u} - 3\vec{v} + \vec{w} = \vec{0}$, so that $\vec{w} = 2\vec{u} + 3\vec{v}$.

35. F; Let $A = B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, for example.

36. T; Matrices A and B can both be transformed into $I = \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$. Running the elementary operations backwards, we can transform I into B (see Example 13 of Section 1.3). Thus we can first transform A into I and then I into B.