

True or False

1. T, by Fact 7.2.2
2. T, by Definition 7.2.3
3. F; If $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, then eigenvalue 1 has geometric multiplicity 1 and algebraic multiplicity 2.
4. T, by Fact 7.4.3
5. T; $A = AI_n = A[\vec{e}_1 \dots \vec{e}_n] = [\lambda_1 \vec{e}_1 \dots \lambda_n \vec{e}_n]$ is diagonal.
6. T; If $A\vec{v} = \lambda\vec{v}$, then $A^3\vec{v} = \lambda^3\vec{v}$.
7. T; Consider a diagonal 5×5 matrix with only two distinct diagonal entries.
8. F, by Fact 7.2.7.
9. T, by Summary 7.1.4
10. T, by Fact 7.2.4
11. F; Consider $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$
12. F; Let $A = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$, $\alpha = 2$, $B = \begin{bmatrix} 4 & 0 \\ 0 & 5 \end{bmatrix}$, $\beta = 5$, for example. Then $\alpha\beta = 10$ isn't an eigenvalue of $AB = \begin{bmatrix} 8 & 0 \\ 0 & 15 \end{bmatrix}$.
13. T; If $A\vec{v} = 3\vec{v}$, then $A^2\vec{v} = 9\vec{v}$.
14. T; Construct an eigenbasis by combining a basis of V with a basis of V^\perp .
15. T, by Fact 7.5.5
16. F; Let $A = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$, for example.
17. T, by Example 6 of Section 7.5
18. T; The geometric multiplicity of eigenvalue 0 is $\dim(\ker A) = n - \text{rank}(A)$.

19. T; If $S^{-1}AS = D$, then $S^T A^T (S^T)^{-1} = D$.
20. F; Let $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, for example.
21. F; Consider $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, with $A^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.
22. T, by Fact 7.5.5
23. F; Let $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$, for example.
24. F; Let $A = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$, with $AB = \begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix}$, for example.
25. T; If $S^{-1}AS = D$, then $S^{-1}A^{-1}S = D^{-1}$ is diagonal
26. F; the equation $\det(A) = \det(A^T)$ holds for all square matrices, by Fact 6.2.1
27. T; The sole eigenvalue, 7, must have geometric multiplicity 3.
28. F; Let $A = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$, with $A + B = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$, for example.
29. F; Consider the zero matrix.
30. T; If $A\vec{v} = \alpha\vec{v}$ and $B\vec{v} = \beta\vec{v}$, then $(A + B)\vec{v} = A\vec{v} + B\vec{v} = \alpha\vec{v} + \beta\vec{v} = (\alpha + \beta)\vec{v}$.
31. F; Consider the identity matrix.
32. T; Both A and B are similar to $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, by Fact 7.4.1
33. F; Let $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, for example.
34. F; Consider $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$
35. F; Let $A = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, and $\vec{w} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, for example.
36. T; A nonzero vector on L and a nonzero vector on L^\perp form an eigenbasis.
37. T; The eigenvalues are 3 and -2 .

38. T, by Fact 7.3.7
39. T, by Fact 7.3.5
40. T; An eigenbasis for A is an eigenbasis for $A + 4I_4$ as well.
41. F; Consider a rotation through $\pi/2$.
42. T; Suppose $\begin{bmatrix} A & A \\ 0 & A \end{bmatrix} \begin{bmatrix} \vec{v} \\ \vec{w} \end{bmatrix} = \begin{bmatrix} A(\vec{v} + \vec{w}) \\ A\vec{w} \end{bmatrix} = \begin{bmatrix} \lambda\vec{v} \\ \lambda\vec{w} \end{bmatrix}$ for a nonzero vector $\begin{bmatrix} \vec{v} \\ \vec{w} \end{bmatrix}$. If \vec{w} is nonzero, then it is an eigenvector of A with eigenvalue λ ; otherwise \vec{v} is such an eigenvector.
43. F; Consider $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$.
44. T; Note that $S^{-1}AS = D$, so that $D^4 = S^{-1}A^4S = S^{-1}0S = 0$, and therefore $D = 0$ (since D is diagonal) and $A = SDS^{-1} = 0$.
45. T; There is an eigenbasis $\vec{v}_1, \dots, \vec{v}_n$, and we can write $\vec{v} = c_1\vec{v}_1 + \dots + c_n\vec{v}_n$. The vectors $c_i\vec{v}_i$ are either eigenvectors or zero.
46. T; If $A\vec{v} = \alpha\vec{v}$ and $B\vec{v} = \beta\vec{v}$, then $AB\vec{v} = \alpha\beta\vec{v}$.
47. T, by Fact 7.3.8
48. F; Let $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, for example.
49. T; Recall that the rank is the dimension of the image. If \vec{v} is in the image of A , then $A\vec{v}$ is in the image of A as well, so that $A\vec{v}$ is parallel to \vec{v} .
50. F; Consider $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$.
51. T; If $A\vec{v} = \lambda\vec{v}$ for a nonzero \vec{v} , then $A^4\vec{v} = \lambda^4\vec{v} = \vec{0}$, so that $\lambda^4 = 0$ and $\lambda = 0$.
52. F; Let $A = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$, for example.
53. T; If the eigenvalue associated with \vec{v} is $\lambda = 0$, then $A\vec{v} = \vec{0}$, so that \vec{v} is in the kernel of A ; otherwise $\vec{v} = A\left(\frac{1}{\lambda}\vec{v}\right)$, so that \vec{v} is in the image of A .
54. T; either there are two distinct real eigenvalues, or the matrix is of the form kI_2 .
55. T; Either $A\vec{u} = 3\vec{u}$ or $A\vec{u} = 4\vec{u}$.
56. T; Note that $(\vec{u}\vec{u}^T)\vec{u} = \|\vec{u}\|^2\vec{u}$.

57. T; Suppose $A\vec{v}_i = \alpha_i\vec{v}_i$ and $B\vec{v}_i = \beta_i\vec{v}_i$, and let $S = [\vec{v}_1 \dots \vec{v}_n]$. Then $ABS = BAS = [\alpha_1\beta_1\vec{v}_1 \dots \alpha_n\beta_n\vec{v}_n]$, so that $AB = BA$.
58. T; Note that a nonzero vector $\vec{v} = \begin{bmatrix} p \\ q \end{bmatrix}$ is an eigenvector of $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ if (and only if) $A\vec{v} = \begin{bmatrix} ap + bq \\ cp + dq \end{bmatrix}$ is parallel to $\vec{v} = \begin{bmatrix} p \\ q \end{bmatrix}$, that is, if $\det \begin{bmatrix} p & ap + bq \\ q & cp + dq \end{bmatrix} = 0$. Check that this is the case if (and only if) \vec{v} is an eigenvector of $\text{adj}(A)$ (use the same criterion).