10. $\lambda_1 = \lambda_2 = 0$ and $\lambda_3 = 9$.

$\vec{v}_1 = \frac{1}{\sqrt{5}}\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$ is in $E_0$ and $\vec{v}_2 = \frac{1}{3}\begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}$ is in $E_9$.

Let $\vec{v}_3 = \vec{v}_1 \times \vec{v}_2 = \frac{1}{3\sqrt{5}}\begin{bmatrix} 2 \\ -4 \\ -5 \end{bmatrix}$; then $\vec{v}_1$, $\vec{v}_2$, $\vec{v}_3$ is an orthonormal eigenbasis.

$$S = \begin{bmatrix} \frac{2}{\sqrt{5}} & \frac{1}{3} & \frac{2}{3\sqrt{5}} \\ \frac{1}{\sqrt{5}} & -\frac{2}{3} & -\frac{4}{3\sqrt{5}} \\ 0 & \frac{2}{3} & -\frac{\sqrt{5}}{3} \end{bmatrix} \text{ and } D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

15. Yes, if $A\vec{v} = \lambda\vec{v}$, then $A^{-1}\vec{v} = \frac{1}{\lambda}\vec{v}$, so that an orthonormal eigenbasis for $A$ is also an orthonormal eigenbasis for $A^{-1}$ (with reciprocal eigenvalues).

19. Let $L(\vec{x}) = A\vec{x}$. Then $A^TA$ is symmetric, since $(A^TA)^T = A^T(A^T)^T = A^TA$, so that there is an orthonormal eigenbasis $\vec{v}_1, \ldots, \vec{v}_n$ for $A^TA$. Then the vectors $A\vec{v}_1, \ldots, A\vec{v}_n$ are orthogonal, since $A\vec{v}_i \cdot A\vec{v}_j = (A\vec{v}_i)^T A\vec{v}_j = \vec{v}_i^T A^T A\vec{v}_j = \vec{v}_i \cdot (A^TA\vec{v}_j) = \vec{v}_i \cdot (\lambda_j \vec{v}_j) = \lambda_j(\vec{v}_i \cdot \vec{v}_j) = 0$ if $i \neq j$.

21. For each eigenvalue there are two unit eigenvectors: $\pm\vec{v}_1$, $\pm\vec{v}_2$, and $\pm\vec{v}_3$. We have 6 choices for the first column of $S$, 4 choices remaining for the second column, and 2 for the third.
Answer: $6 \cdot 4 \cdot 2 = 48$.