

Math 21b
April 20, 2000

Exercises for Chapter 9

65. Which of the following sets are subspaces of C^∞ ? Justify your answers.

- (i) All continuous functions from \mathbf{R} to \mathbf{R} .
- (ii) All $f \in C^\infty$ such that $f(0) + f'(0) = 0$.
- (iii) All $f \in C^\infty$ such that $f + f' \equiv 0$.
- (iv) All $f \in C^\infty$ such that $f(0) = 1$.

66. Which of the following subsets of C^∞ are linearly independent? Justify your answers.

- (i) $1, t, t^2, t^3$
- (ii) $1 + t, 1 - t, t^2, 1 + t + t^2$
- (iii) $\sin t, e^t, e^{-t}$
- (iv) $\sin t, \cos t, \sin(t + \frac{\pi}{3})$

67. Which of the following functions are linear? Justify your answers.

- (i) $T : C^\infty \rightarrow \mathbf{R}; T(f) = f(0)$
- (ii) $T : C^\infty \rightarrow C^\infty; T(f) = f^2 + f'$
- (iii) $T : C^\infty \rightarrow \mathbf{R}^2; T(f) = \begin{bmatrix} f(0) \\ f(1) \end{bmatrix}$
- (iv) $T : C^\infty \rightarrow \mathbf{R}; T(f) = \int_0^1 f(t) dt$

68. Find a basis for the kernel of $T : C^\infty \rightarrow C^\infty$ given by

$$T(f)(t) = f''(t) - f(0).$$

69. Find a basis for the image of $T : C^\infty \rightarrow C^\infty$ given by

$$T(f)(t) = f(0) + f'(0)t + [f(0) + f'(0)]t^2.$$

54. Find the eigenvalues and eigenspaces for $T : C^\infty \rightarrow C^\infty$ given by

$$T(f) = f + f'.$$