MATH 21B REVIEW

Jan 13, 2004

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TECHNOLOGY

Does PowerPoint make us stupid?

...int into art

...42 GMT)

executives.

But after spending several hours

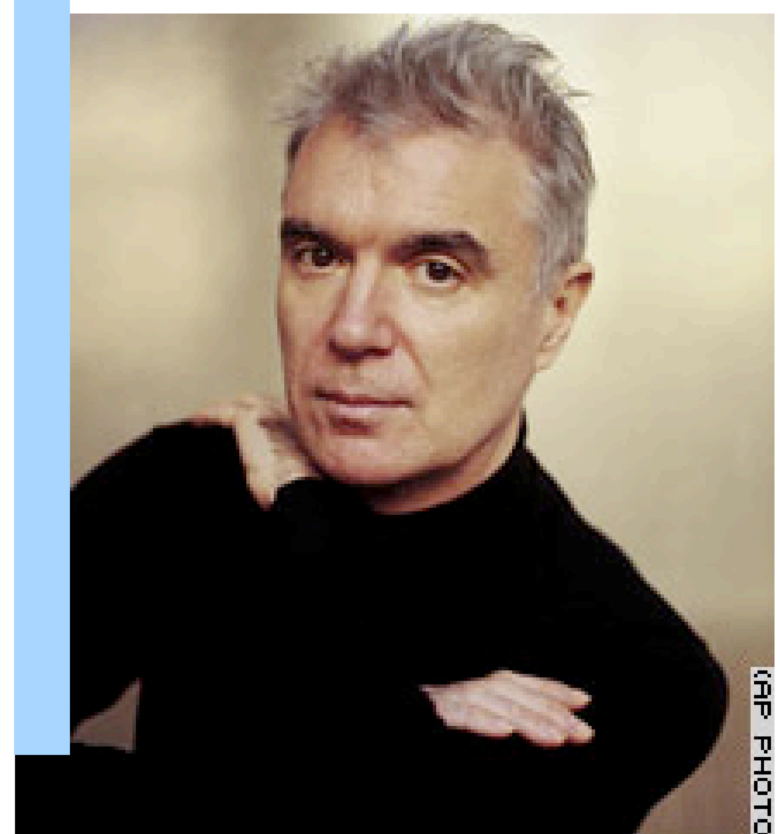
Less Tangible

It's the PowerPoint, Stupid

31 December 2003
by Marc Zeedar
Contributing Columnist

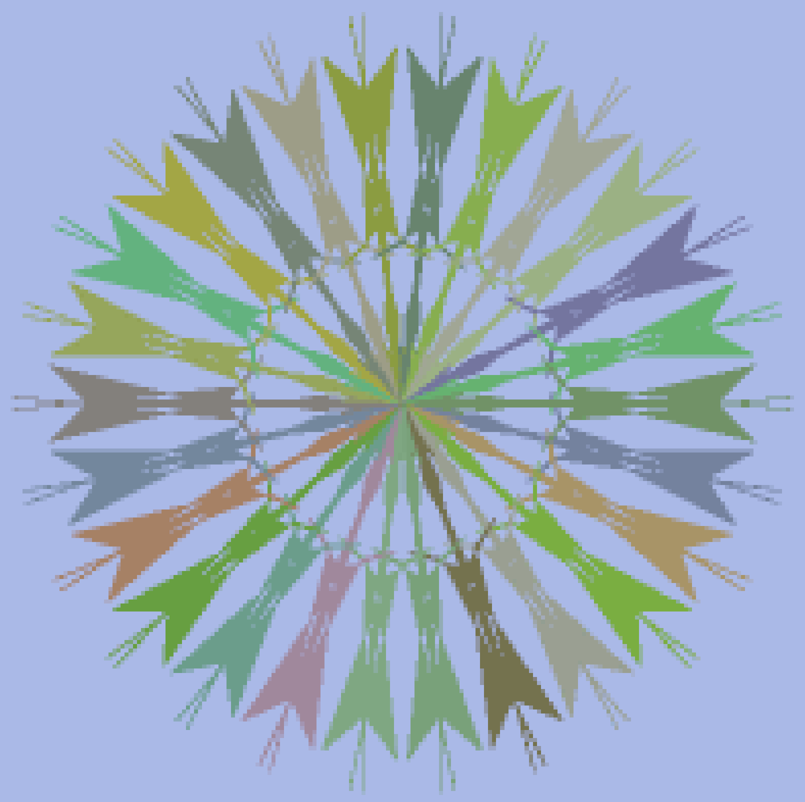
print format | email this story | talk

The *New York Times* recently ran an article titled PowerPoint Makes You Dumb. In it, the author cites sources who blame the Columbia space shuttle disaster on Microsoft's presentation software. You see, apparently those guys at NASA used a PowerPoint slide to "explain"



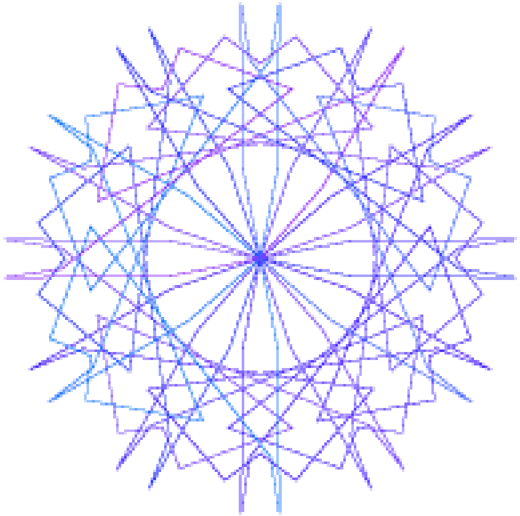
David Byrne, a composer, photographer and

I) Preliminaries



- Diagonalization
- Complex Numbers
- Linear spaces, linear transformations
- Differential operators

Diagonalization



Diagonalization of a matrix A is possible if:

Orthonormal eigenbasis

- A is a symmetric matrix
- All eigenvalues of A are different

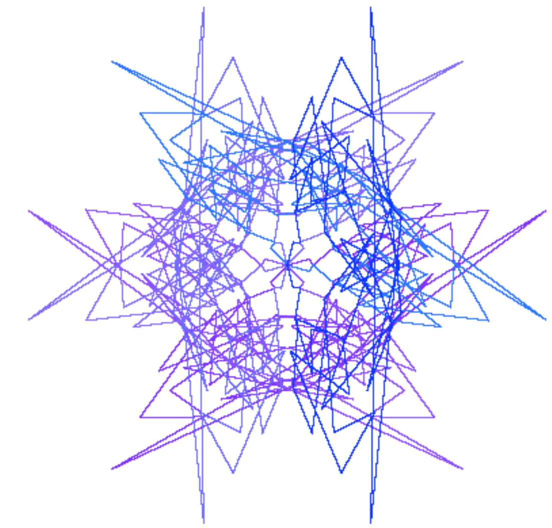
Prototype of nondiagonalizable matrix :

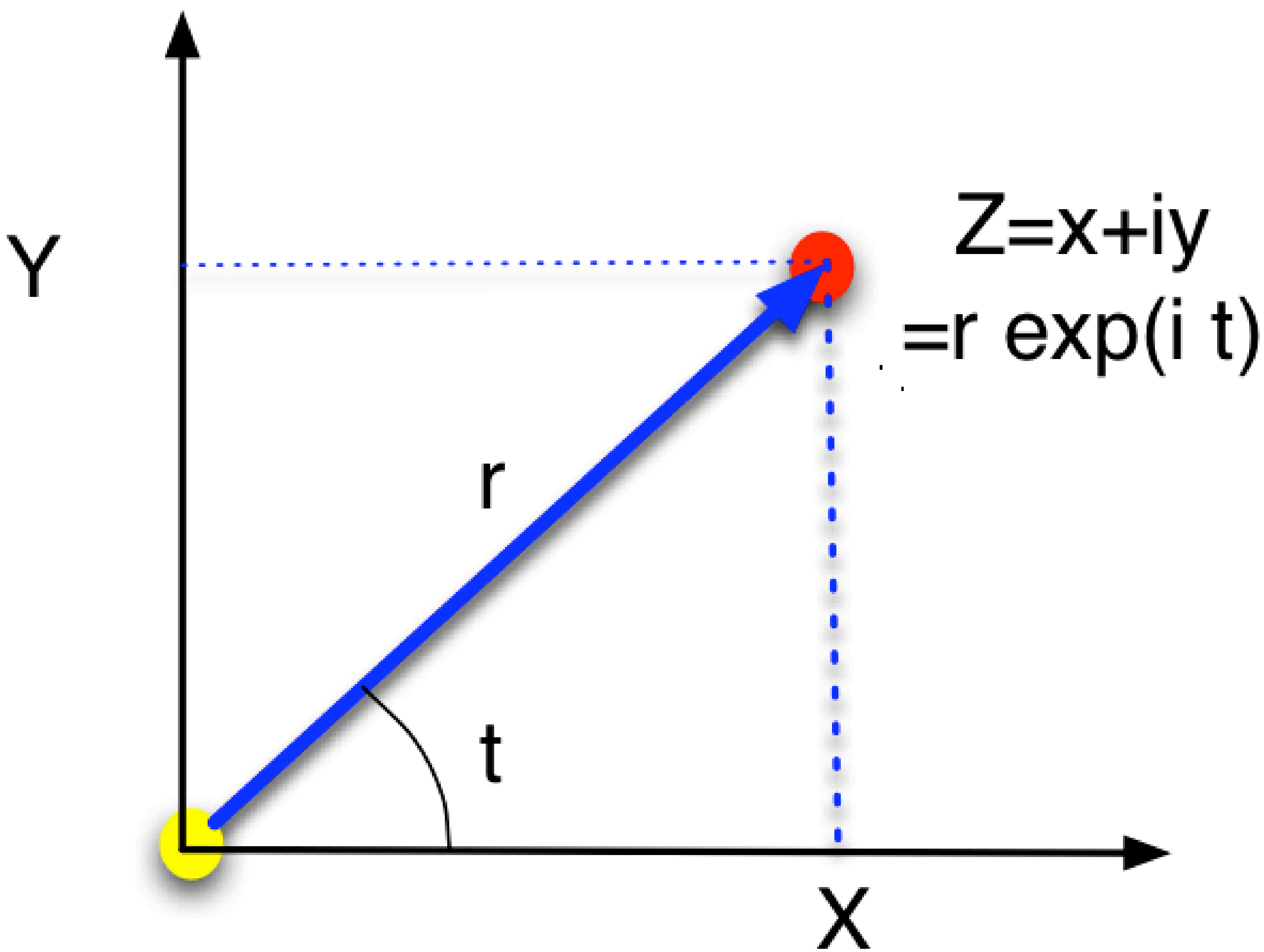
$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

Jordan Normal Form

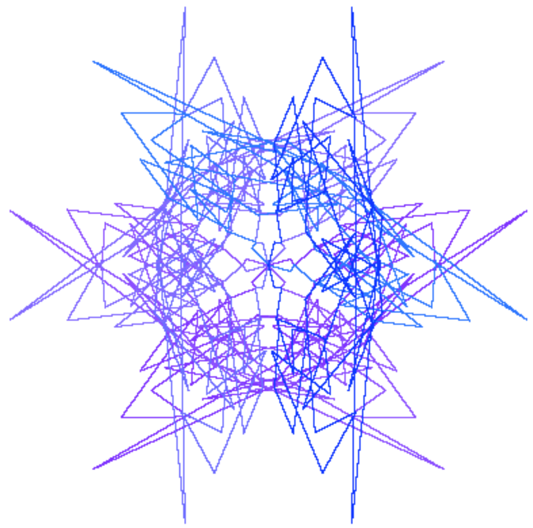
$$\begin{bmatrix} \lambda & 1 & 0 & 0 & 0 \\ 0 & \lambda & 1 & 0 & 0 \\ 0 & 0 & \lambda & 1 & 0 \\ 0 & 0 & 0 & \lambda & 1 \\ 0 & 0 & 0 & 0 & \lambda \end{bmatrix}$$

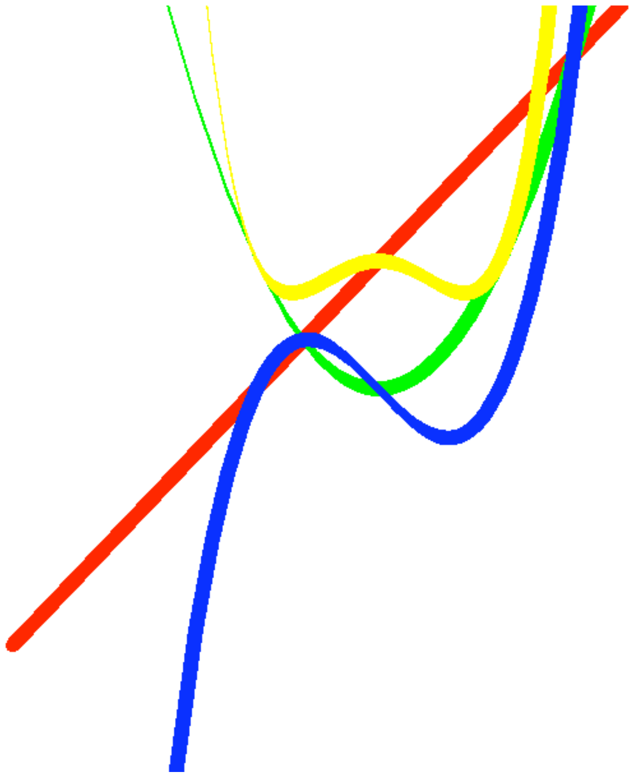
Complex Numbers





Fundamental Theorem of algebra

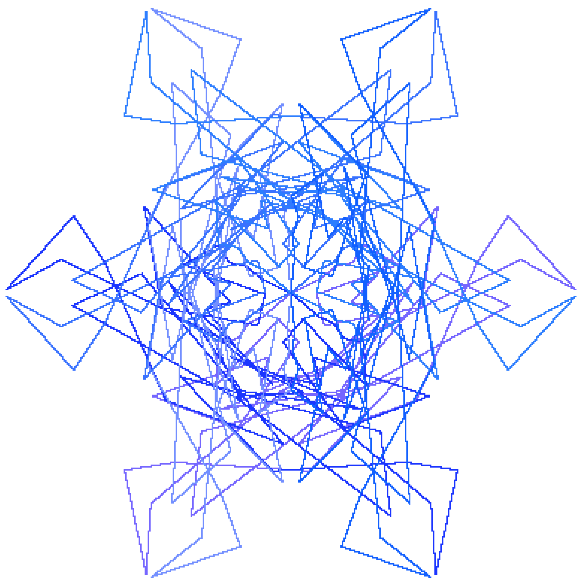


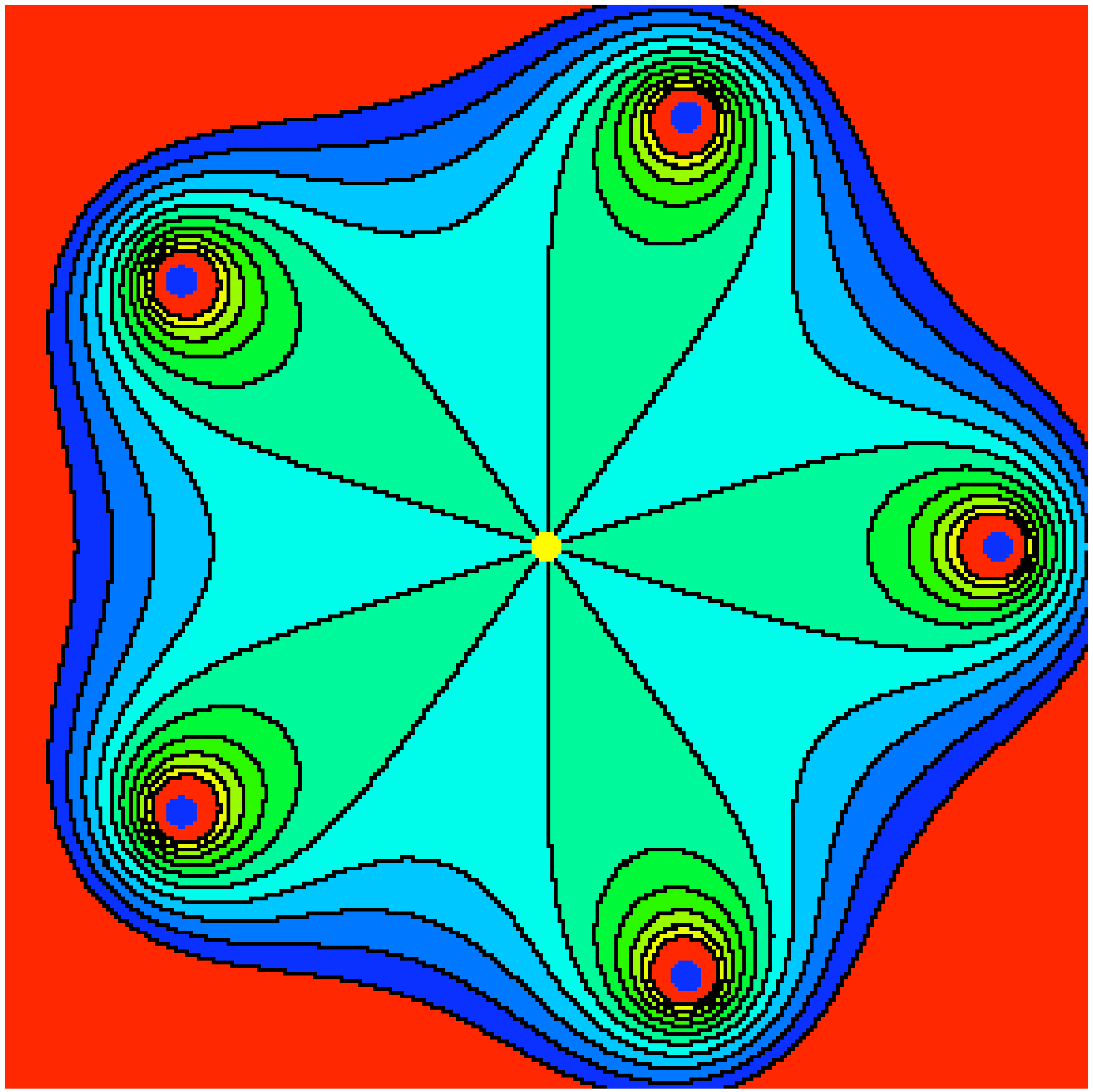


A polynomial of degree n has exactly n roots.

- If a+ib is a root, then a-ib is a root too.
- If n is odd, there is at least one real root.

Roots of One





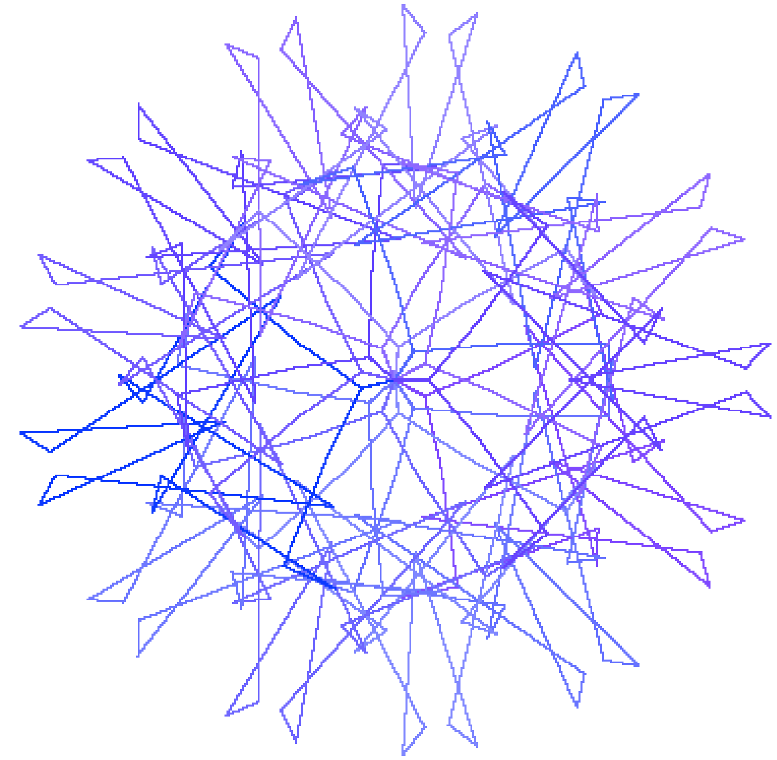
Example:
permutation matrix:

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$p(\lambda) = (\lambda^n - 1) = (\lambda - \lambda_1)(\lambda - \lambda_2)\ldots(\lambda - \lambda_n)$$

Linear Spaces

In a linear space, we can add and scale.

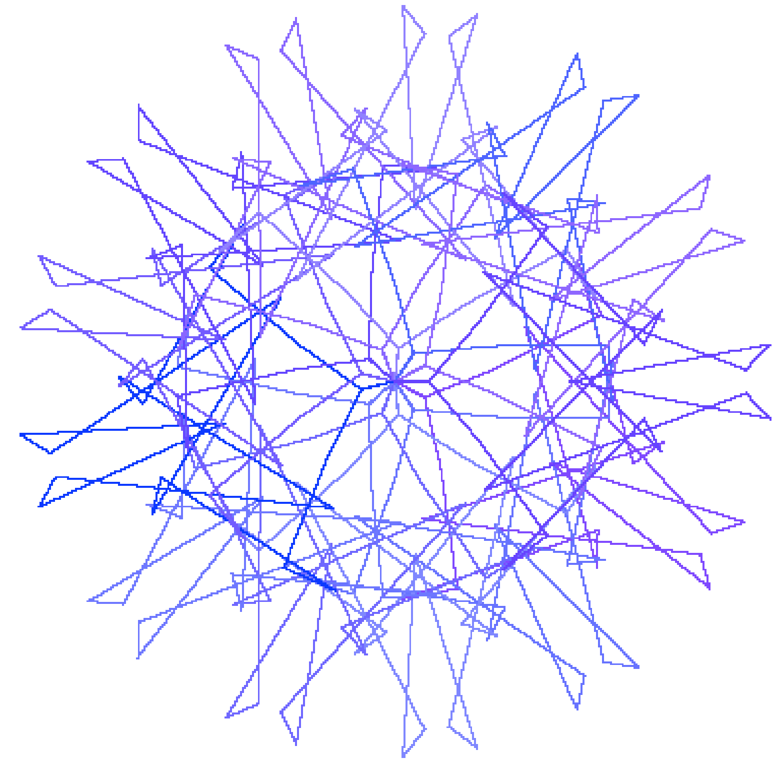


$\mathbf{R}^n$	$\mathbf{M}(\mathbf{R}, n)$
$C^\infty(\mathbf{R})$	$C^\infty([-\pi, \pi])$

To check whether a subset X of one of these spaces is a linear space, we check:

x+y is in X	rx is in X	0 is in X

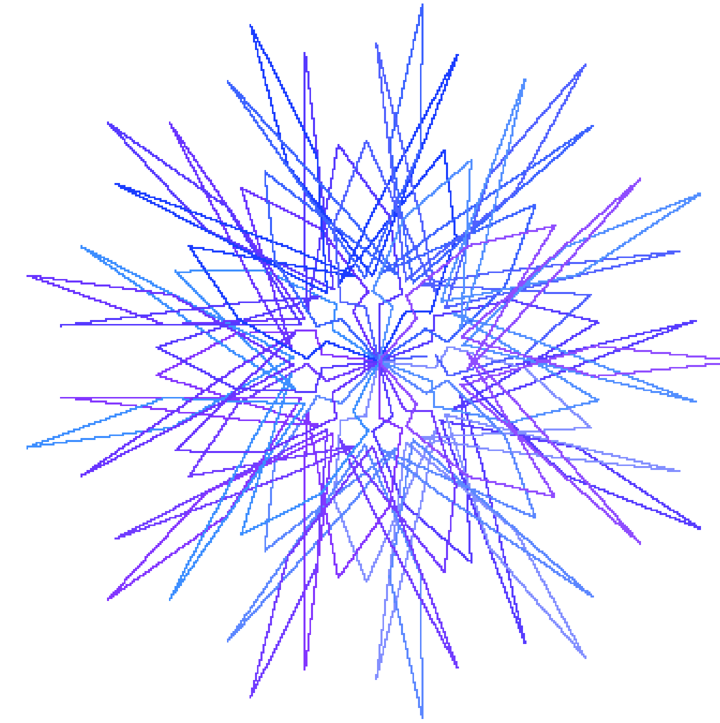
Linear Maps



To check wether a map between linear spaces is linear, we have to check:

$$T(x+y) = T(x)+T(y), \quad T(rx)=r\,T(x), \quad T(0)=0$$

Differential operators



Df=f'

p(x) polynomial

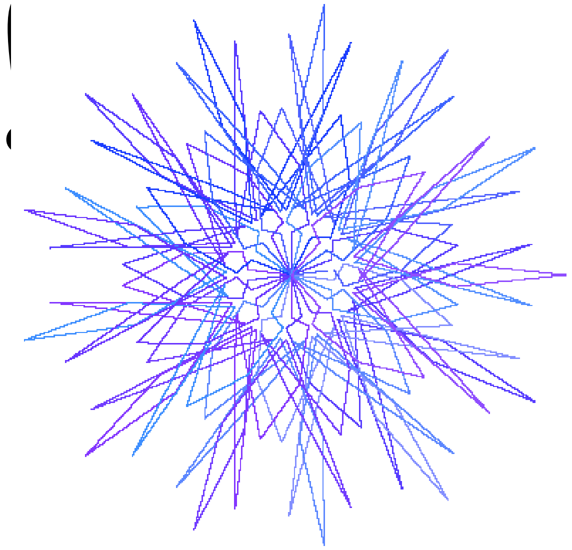
T=p(D) differential operator

T f = g differential equation

Fundamental theorem of calculus:

$$p(D) = (D - \lambda_1)(D - \lambda_2)...(D - \lambda_n)$$

This is a lot of material!



“We like it extreme”

Quizz coming up!

Win

Structure of the quizz:

2)		2)	Shout:
3)	True	3)	21 14
5)		5)	
7)		7)	
11)		11)	

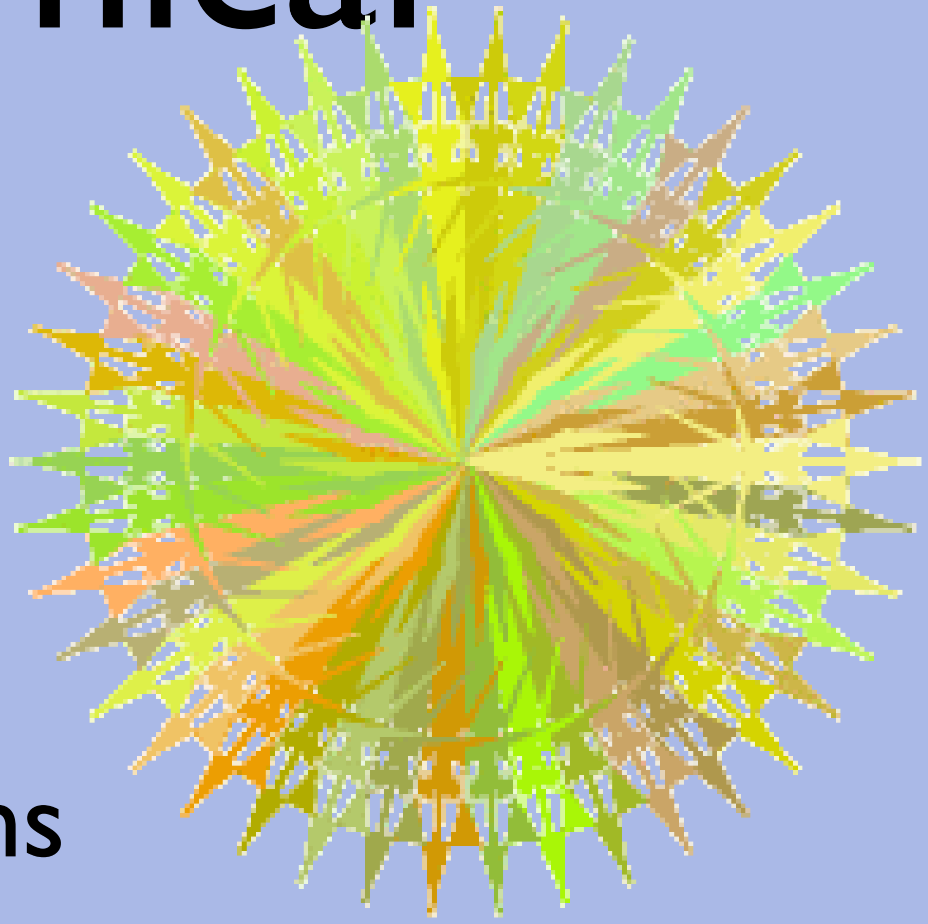
Linear space or not?

2) Smooth 2π-periodic functions with $\int_0^{2\pi} f(x)\,dx = 0$.
3) $\{ f \in C^\infty(\mathbf{R}) \mid f(10) = 1 \}$
5) Smooth 2π-periodic functions satisfying $f'(0) = 0$.
7) 2×2 matrices satisfying $\mathrm{tr}(A) = 0$.
11) 2×2 matrices satisfying $\det(A) = 0$.

Linear map or not?

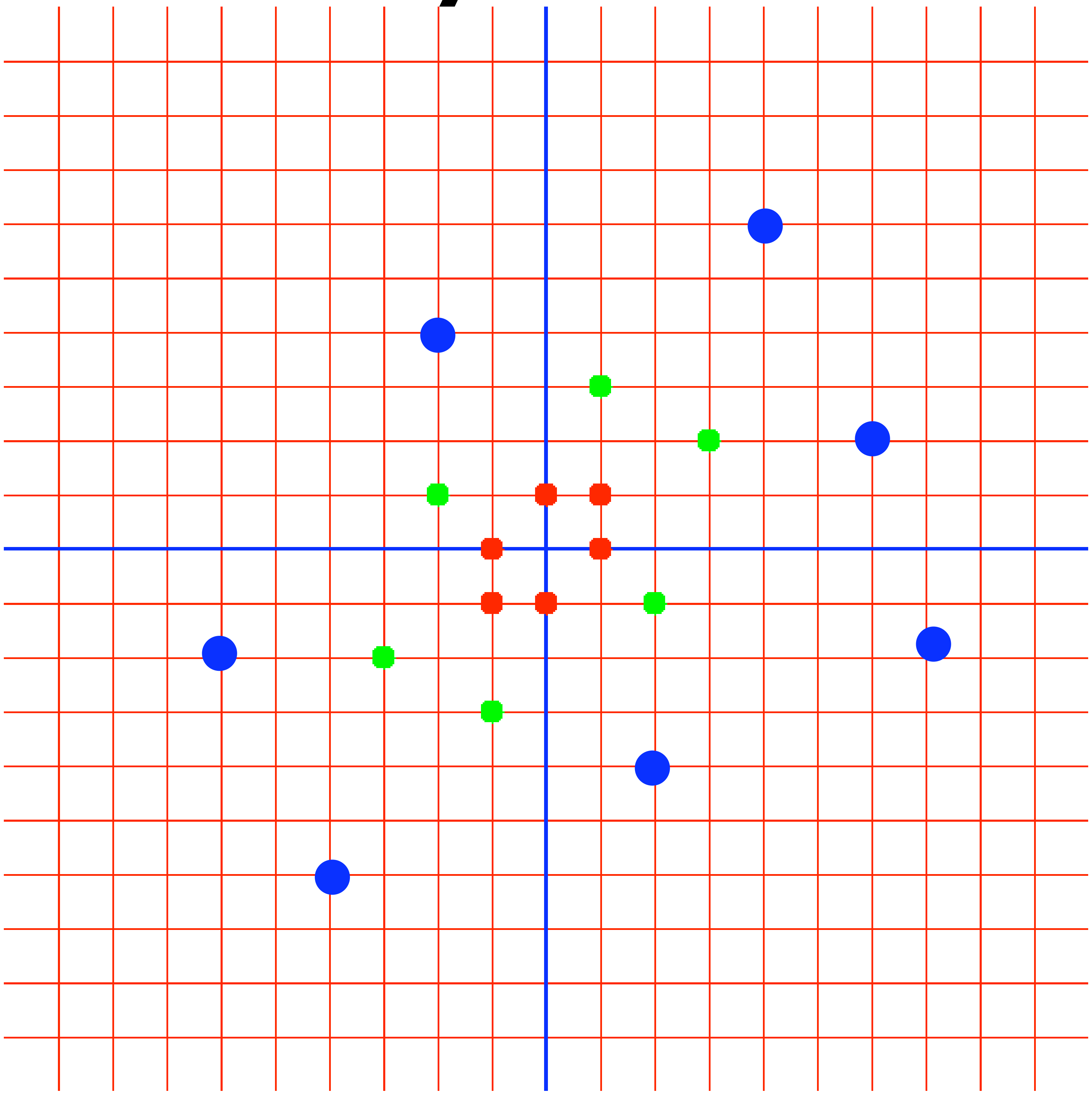
2) $T(f)(x) = x^2 f(x)$.
3) $T(f)(x) = f(1)^2 + f(x)$.
5) $T(f)(x) = f'(x)$.
7) $T(f)(x) = f(x) f'(x)$.
11) $T(f)(x) = x + f(x)$.

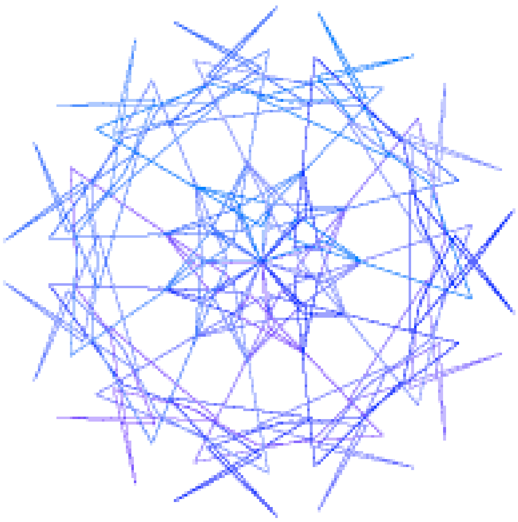
II) Discrete Dynamical Systems



- Solving initial value problems
- Analyse phase space
- Find out about stability

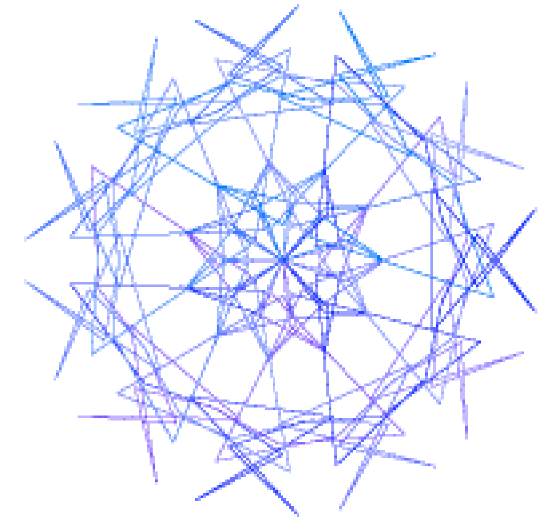
Dynamics



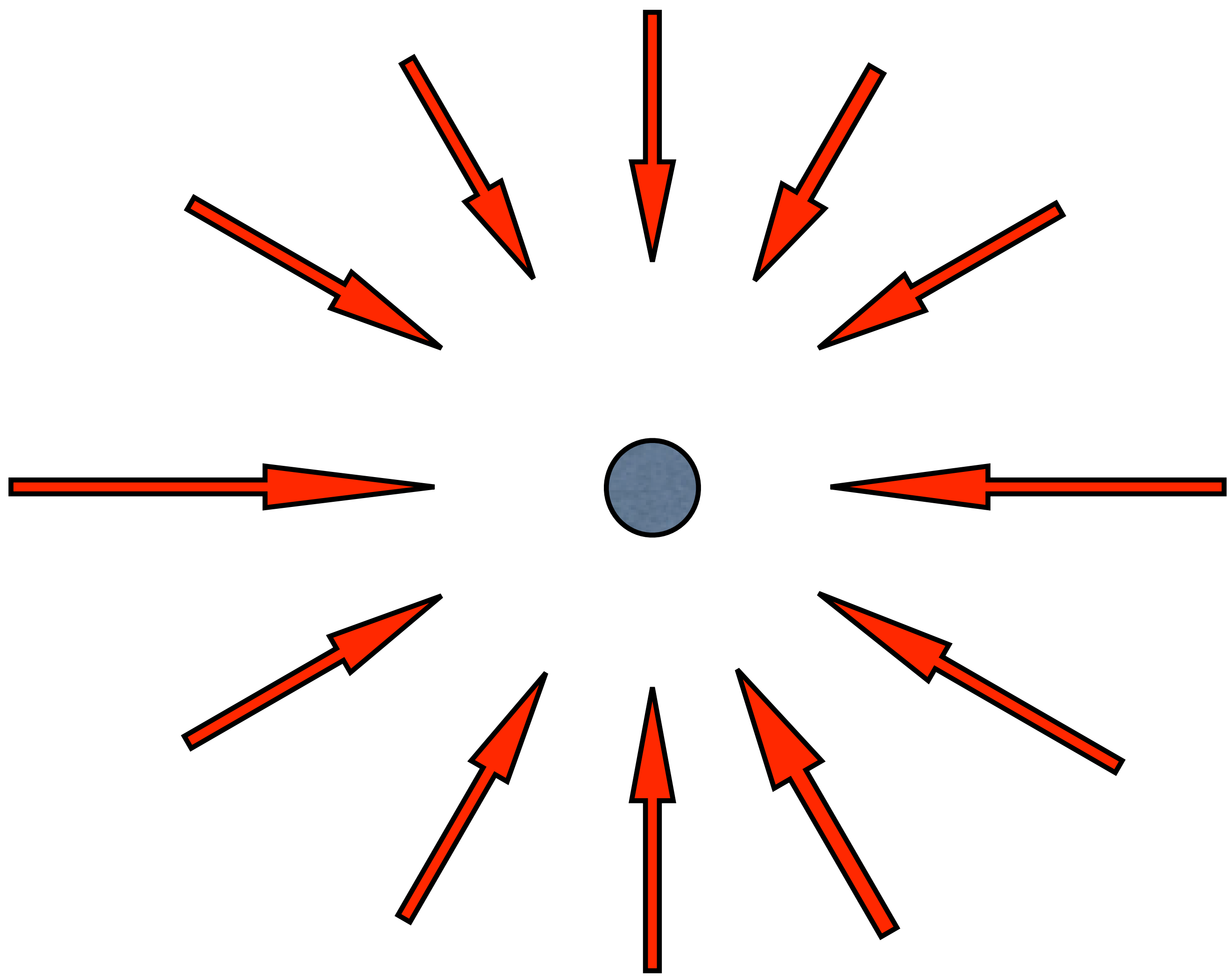


$$A = \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}$$

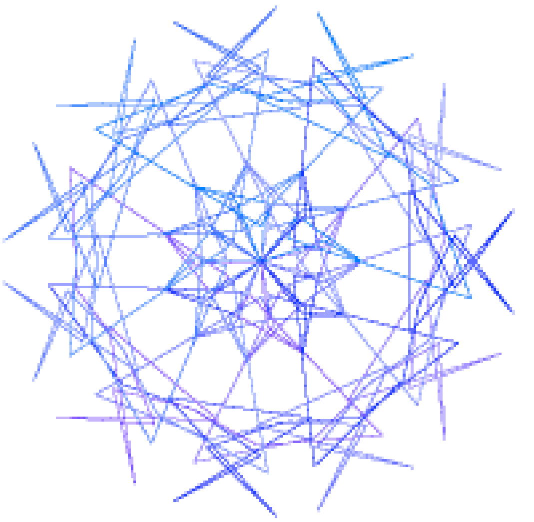
Asymptotic Stability



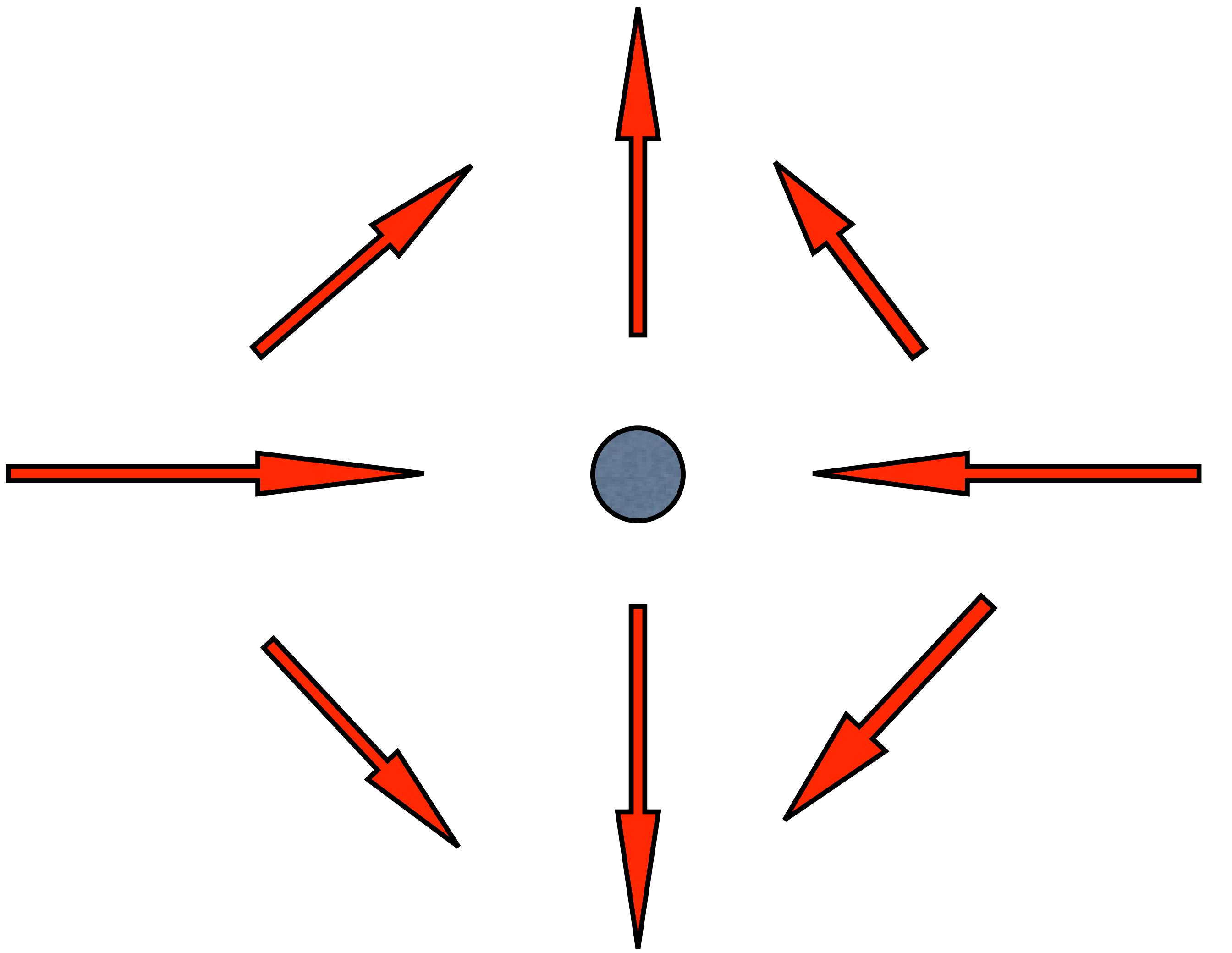
$$A\vec{x} = \begin{bmatrix} 1/4 & 0 \\ 0 & 1/2 \end{bmatrix} \vec{x}$$



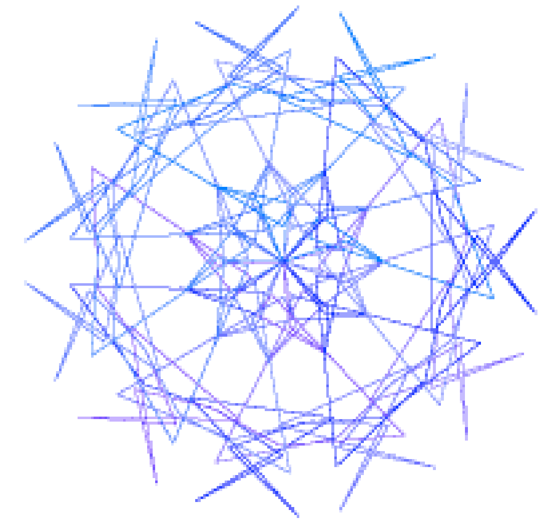
Hyperbolic Behavior



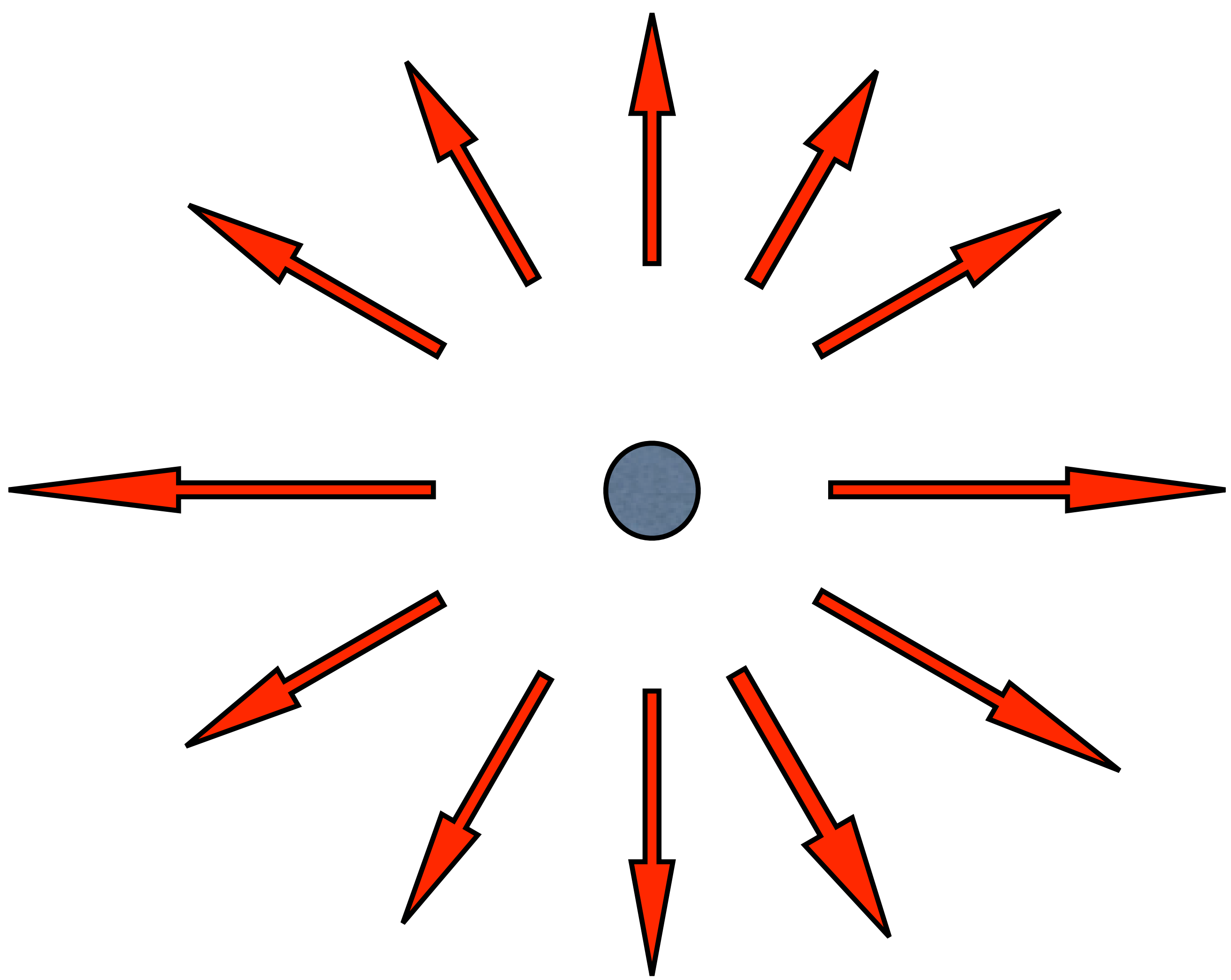
$$A\vec{x} = \begin{bmatrix} 1/2 & 0 \\ 0 & 2 \end{bmatrix} \vec{x}$$



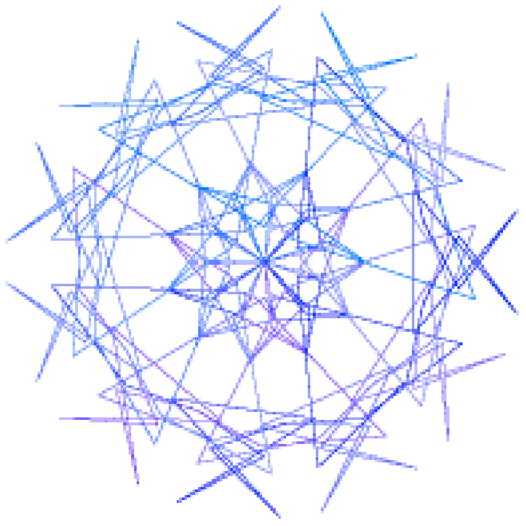
Expansive behavior



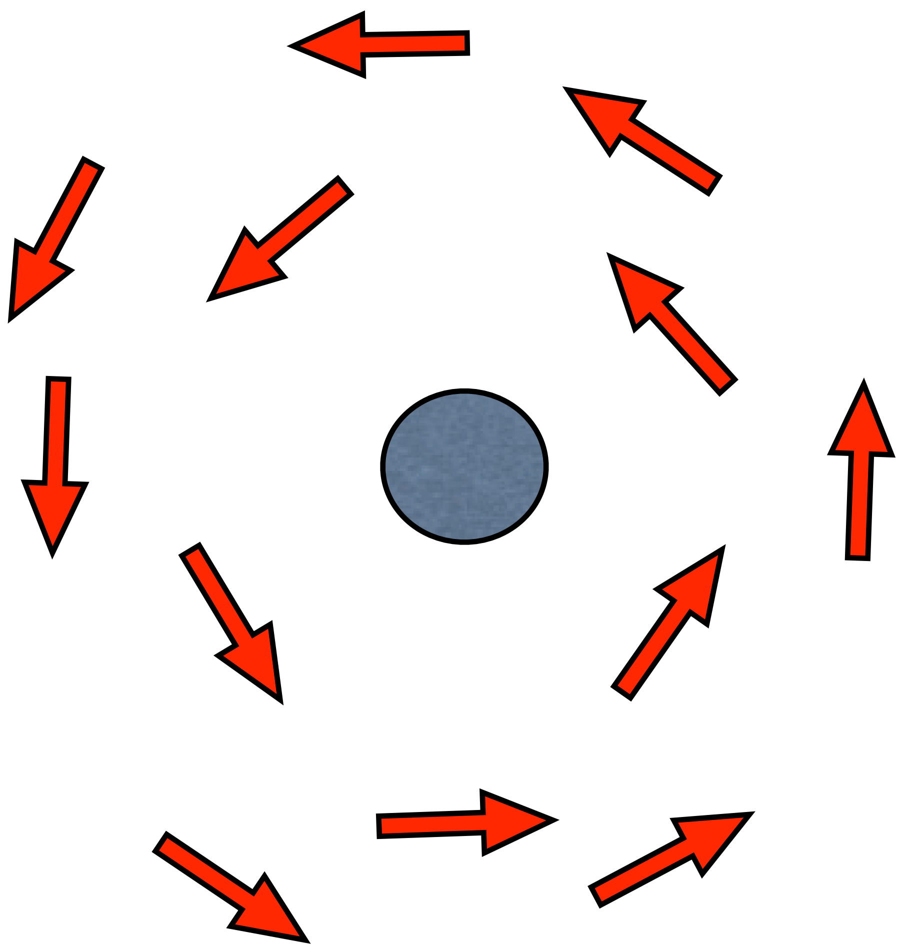
$$A\vec{x} = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \vec{x}$$



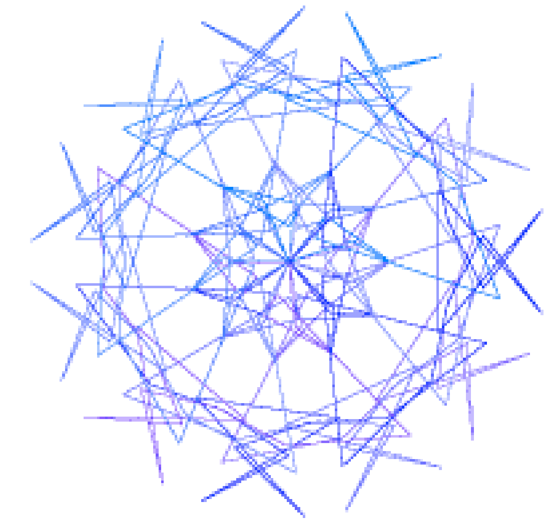
Rotational Behavior



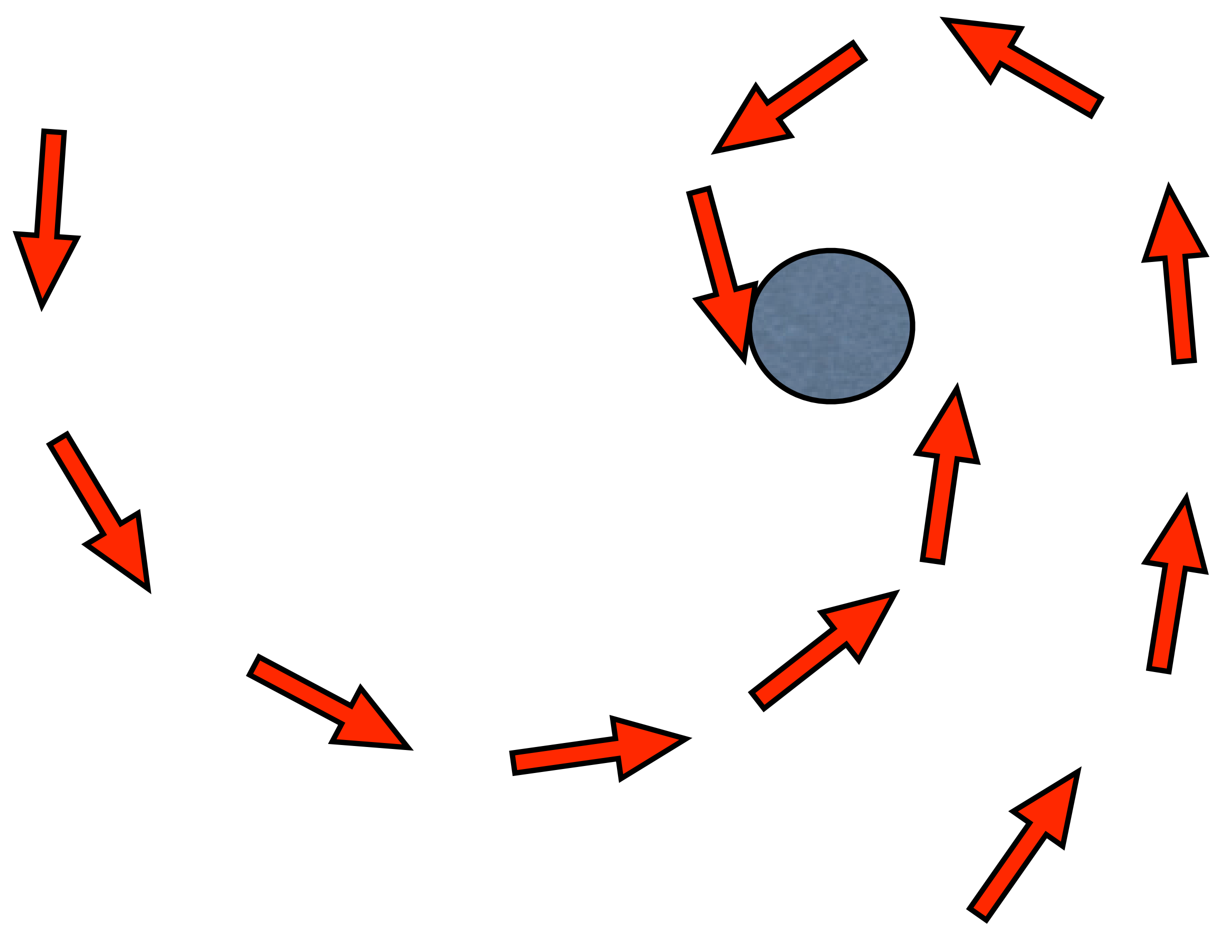
$$A\vec{x} = \begin{bmatrix} \cos(\alpha) & \sin(\alpha) \\ -\sin(\alpha) & \cos(\alpha) \end{bmatrix} \vec{x}$$



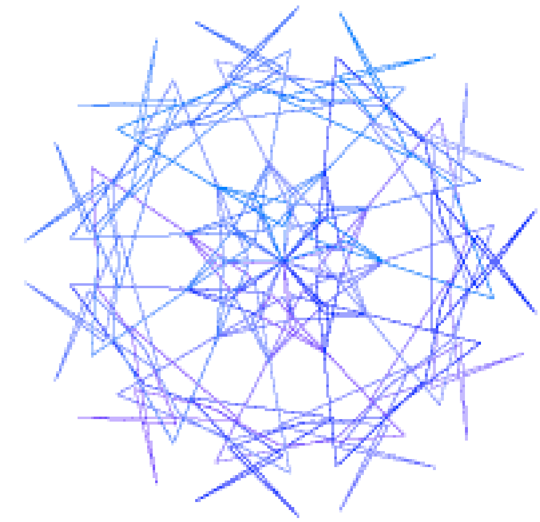
Rotation Dilation



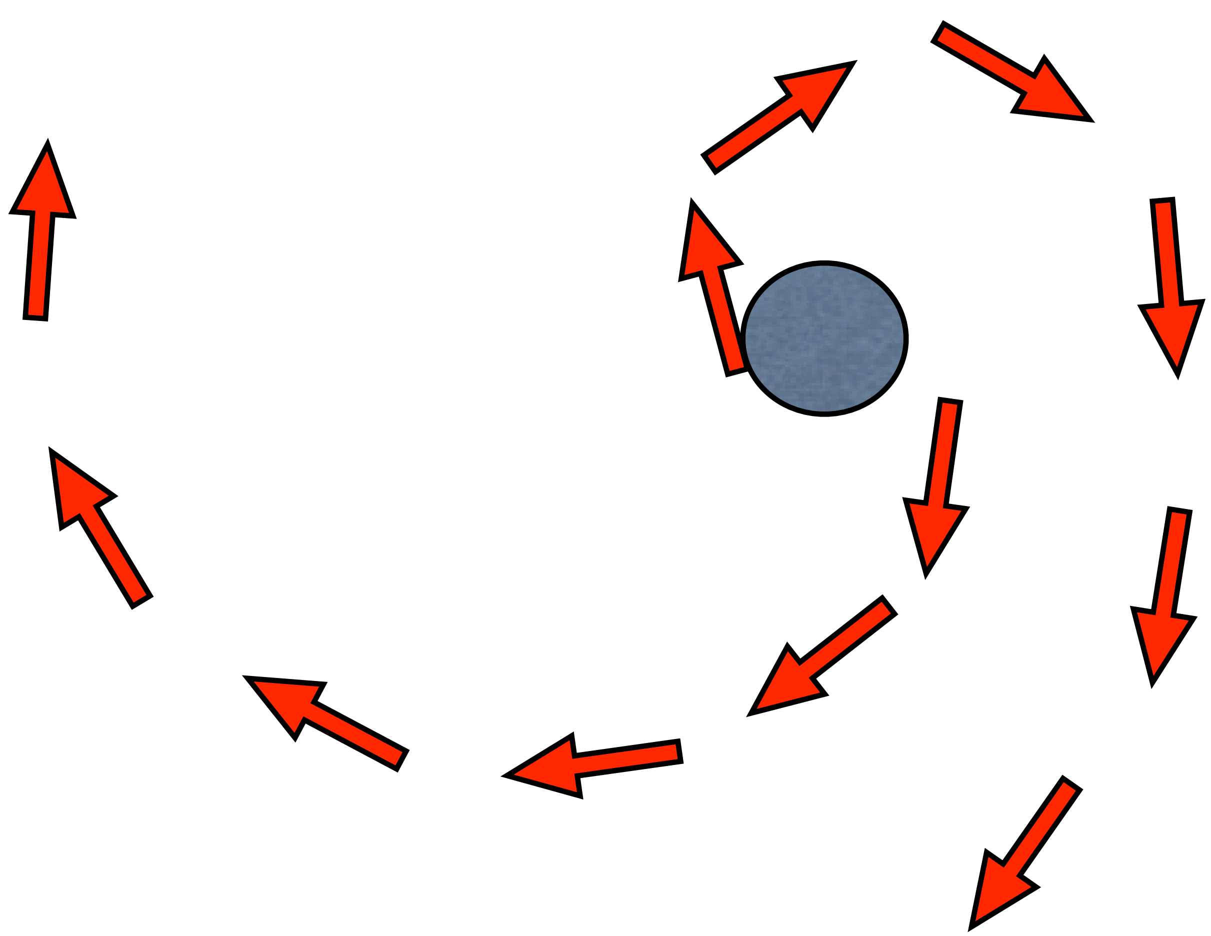
$$A\vec{x} = \frac{1}{2}\begin{bmatrix} \cos(\alpha) & \sin(\alpha) \\ -\sin(\alpha) & \cos(\alpha) \end{bmatrix}\vec{x}$$



Rotation Dilation



$$A\vec{x} = 2\begin{bmatrix} \cos(\alpha) & \sin(\alpha) \\ -\sin(\alpha) & \cos(\alpha) \end{bmatrix}\vec{x}$$



Eigenvalues

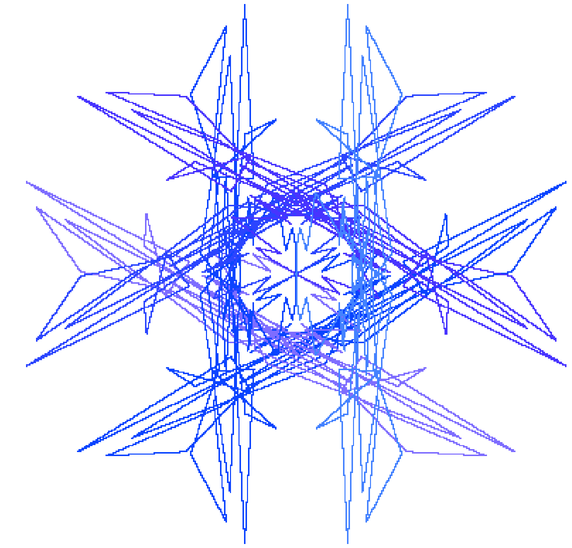
The eigenvalues of A determine the stabilty of the origin.

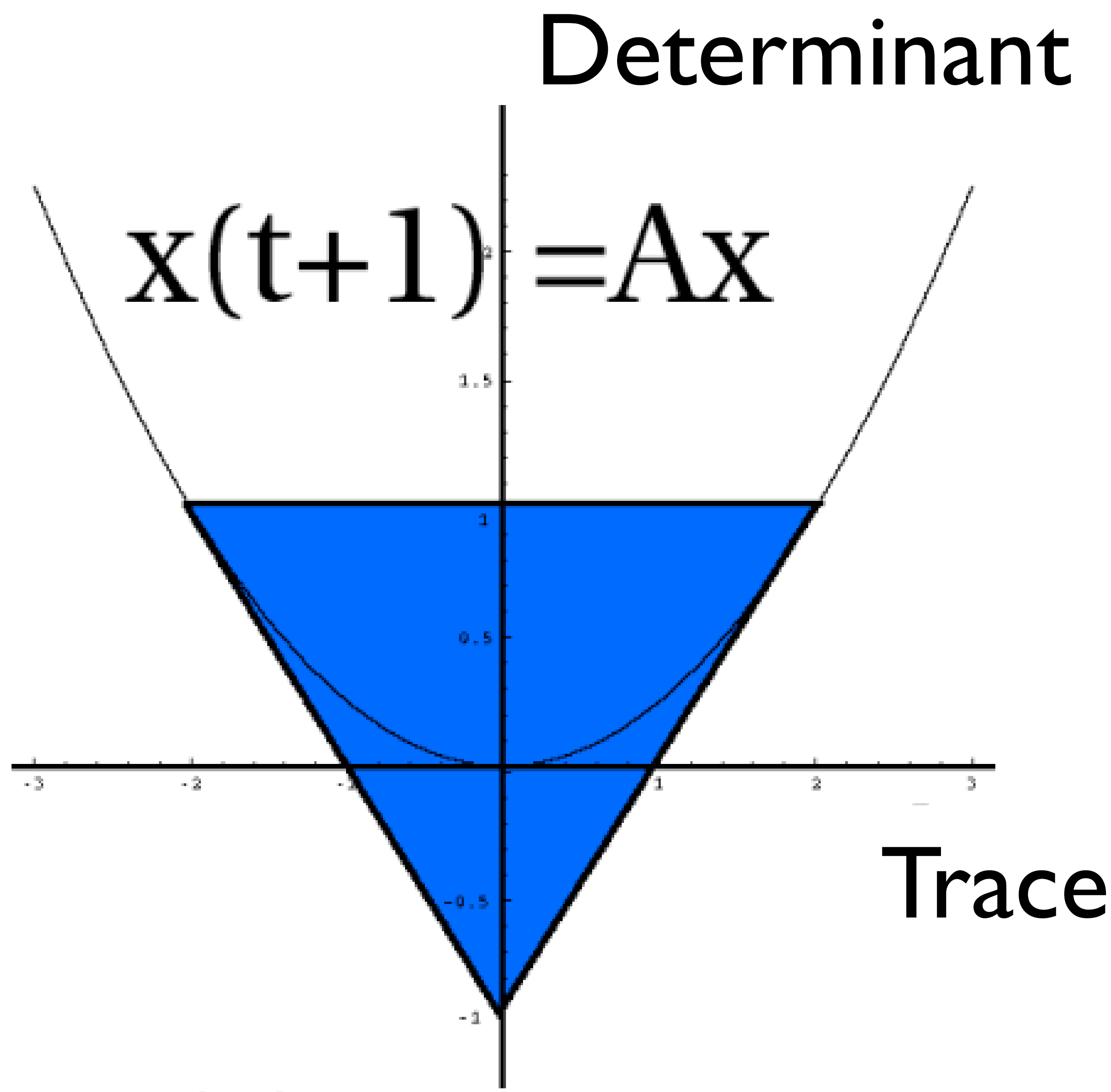
Asymptotic stability

$\Updownarrow$

All eigenvalues have absolute value <1

Stability for discrete systems





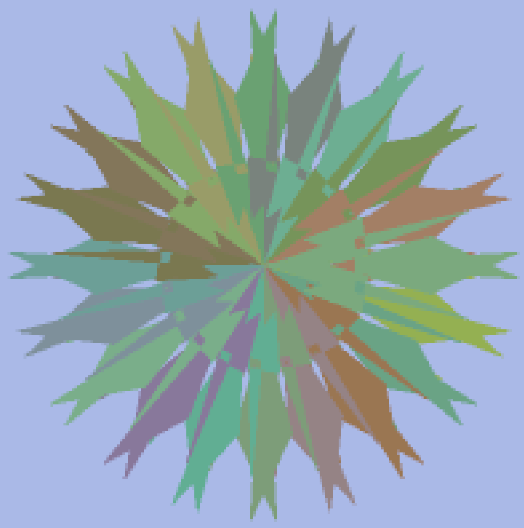
$|tr(A)|-1<det(A)<1$

Initial Value Problem

- Diagonalize A
- Write x as sum of eigenvector
- Write down solution

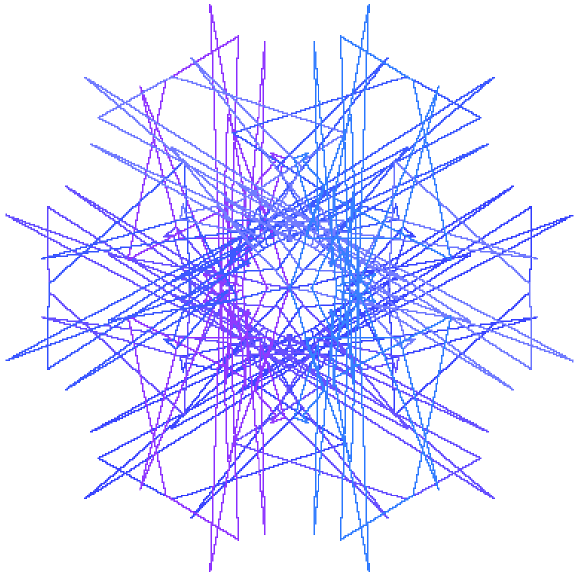
Example Blackboard

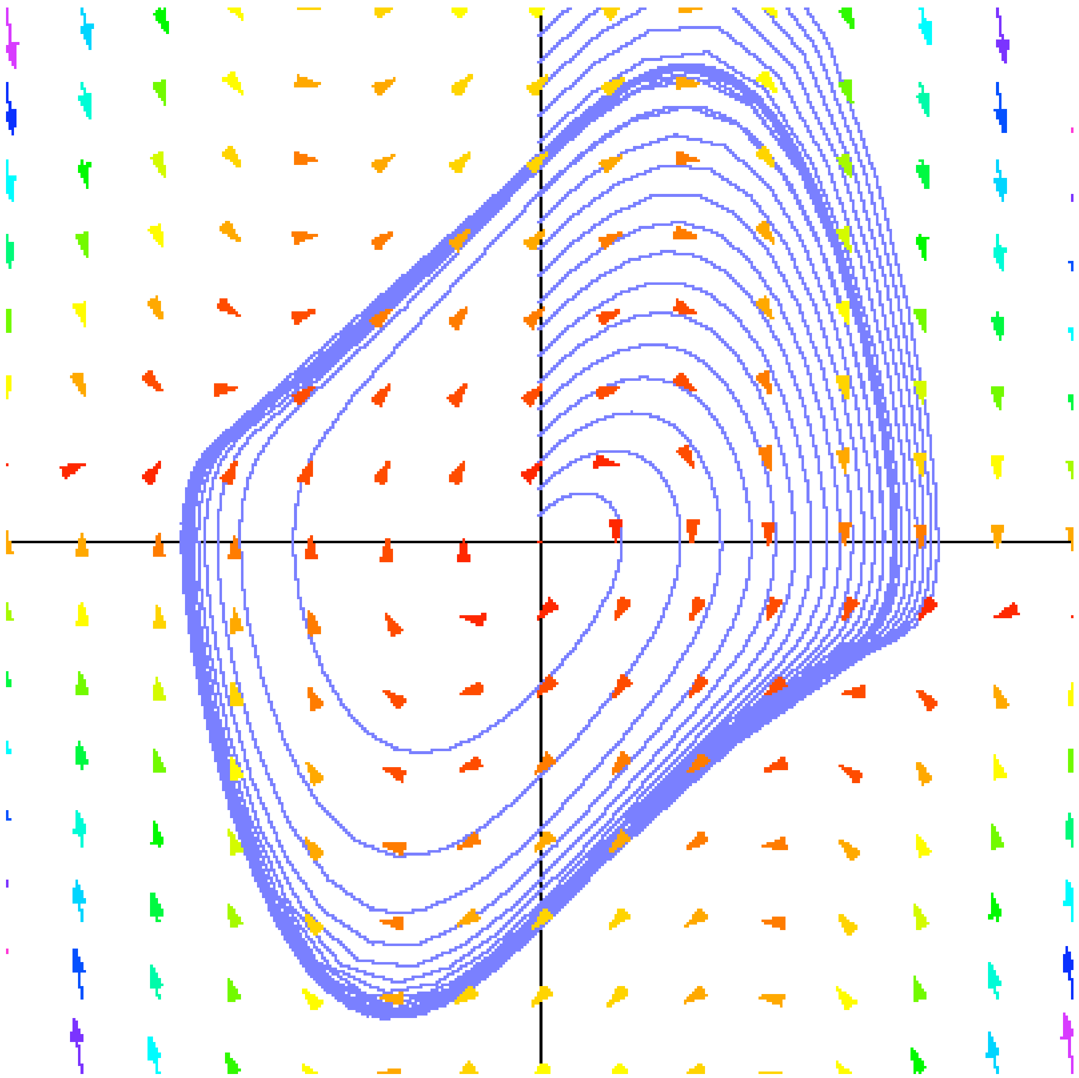
III) Differential Equations



- Solving
- Phase space
- Stability

Differential equations

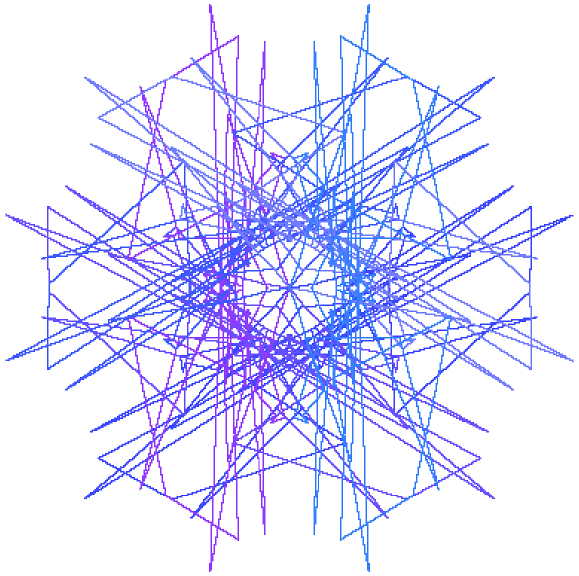




$$\dot{x} = f(x,y)$$

$$\dot{y} = g(x,y)$$

Linear Systems



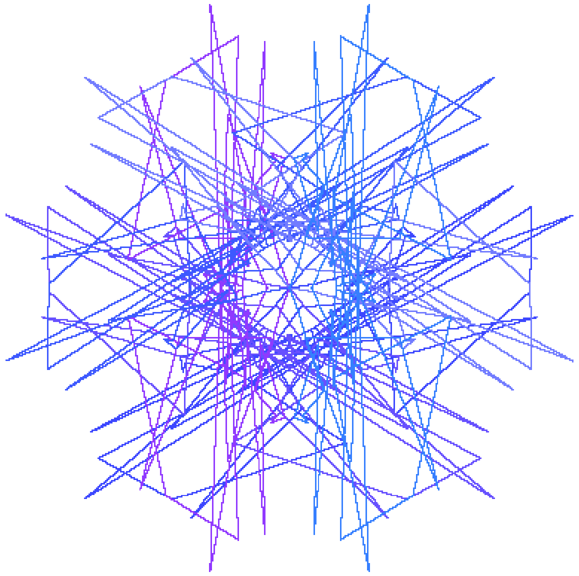
$\dot{x} = ax + by$

$\dot{y} = cx + dy$

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

The eigenvalues of A determine the dynamical behavior.

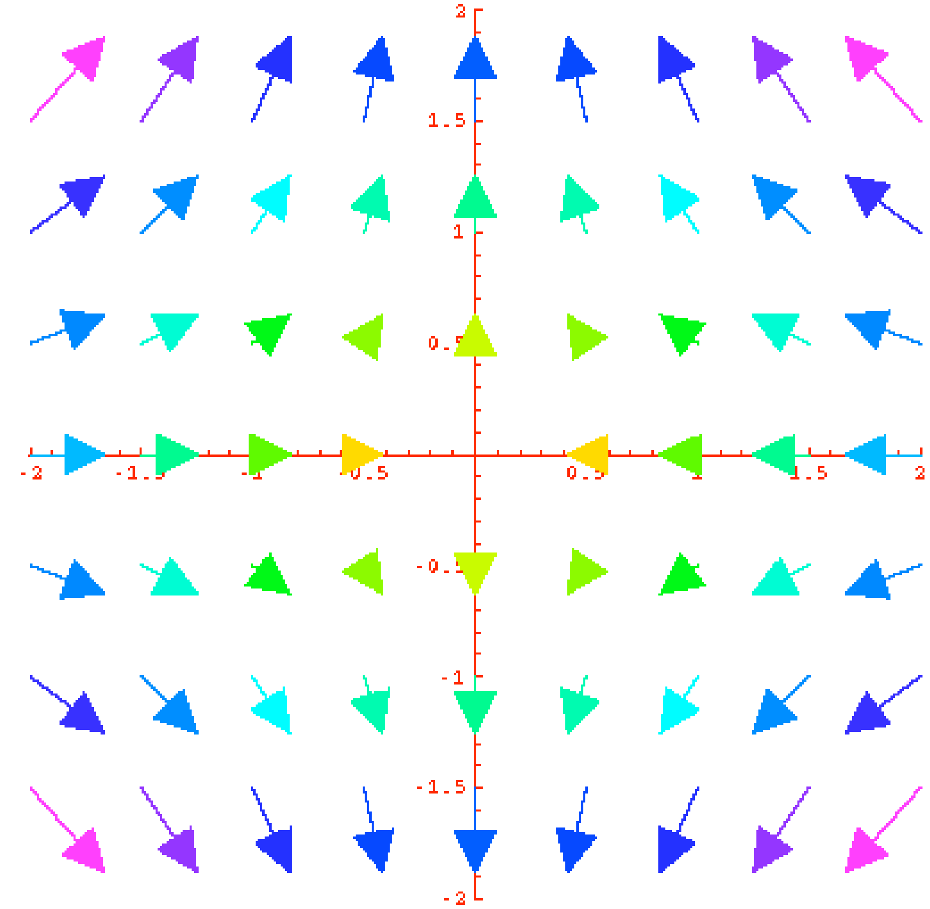
1D Case

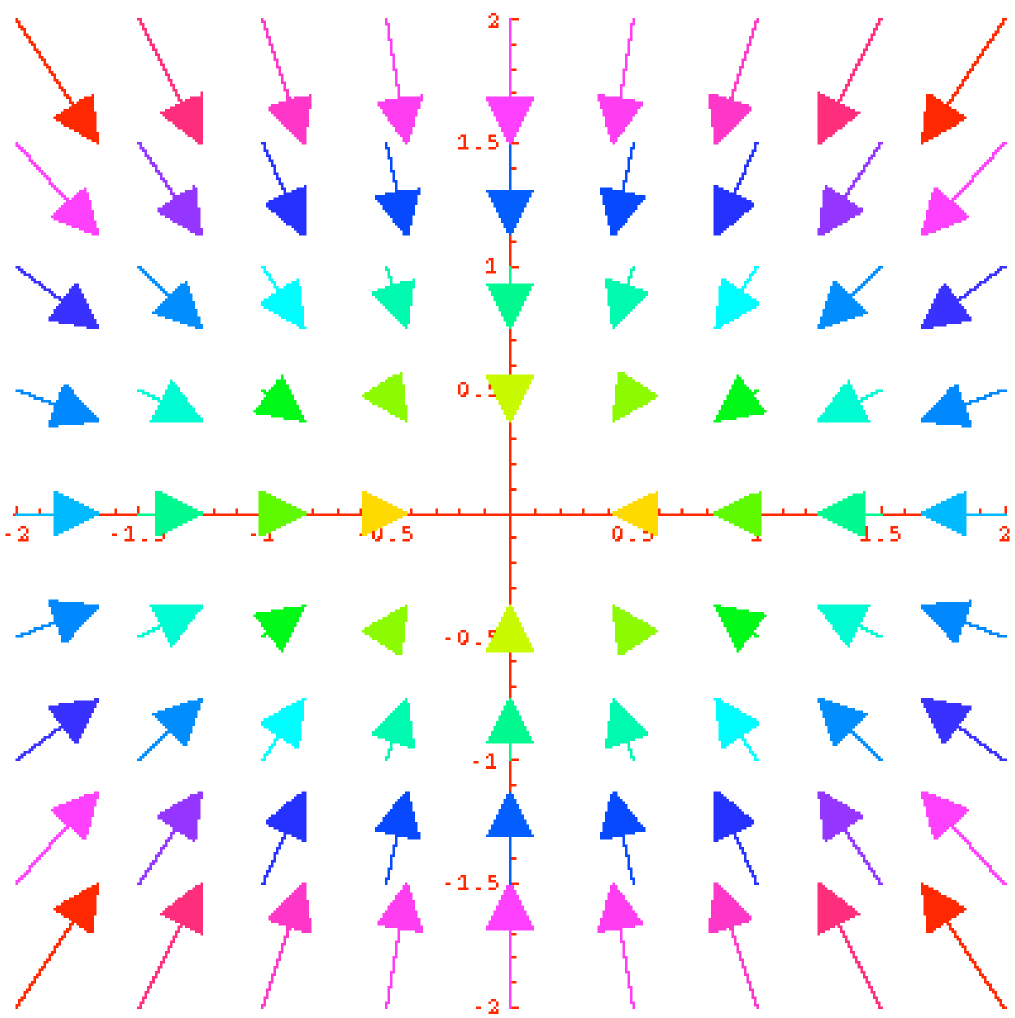


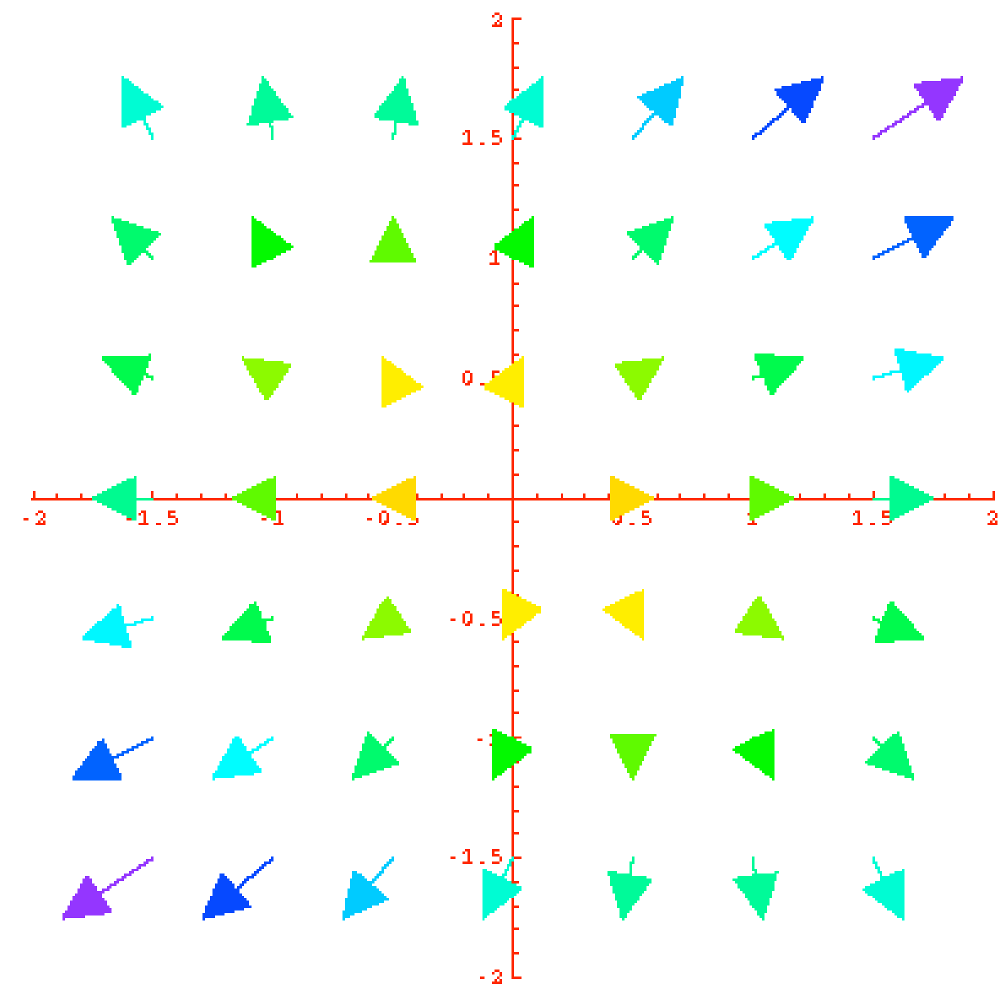
$$\frac{d}{dt}x = \lambda x$$

$$x(t) = e^{\lambda t}x(0)$$

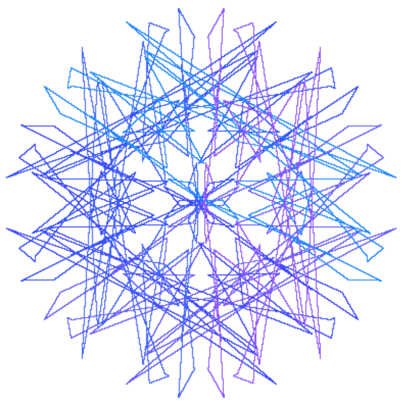
Real, nonzero eigenvalues

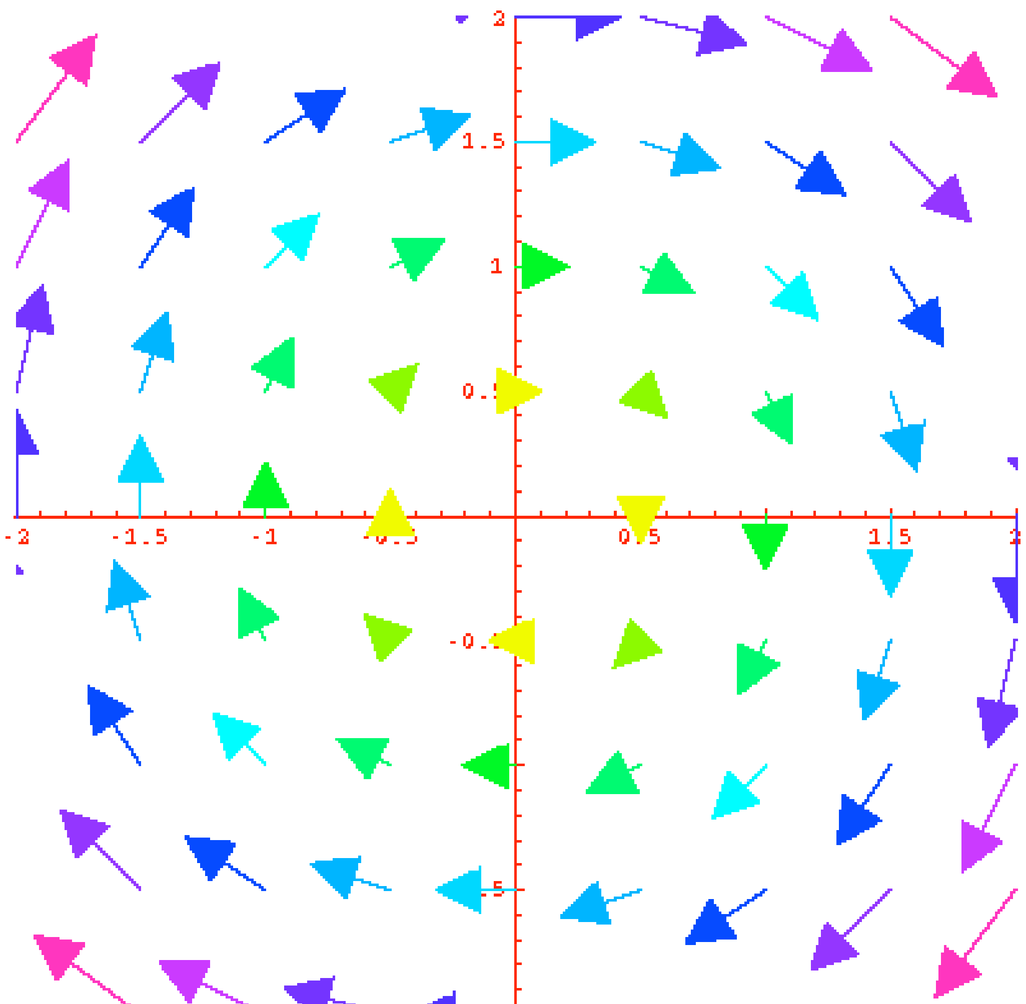


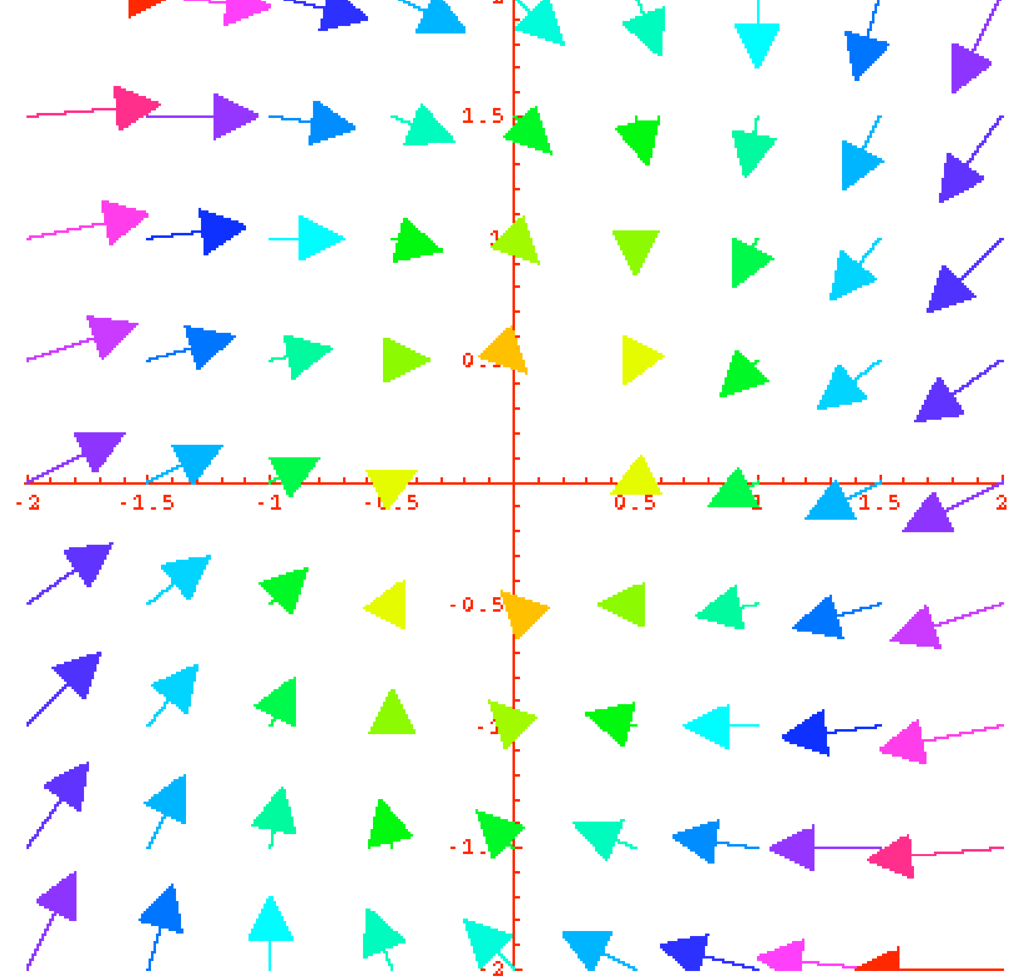


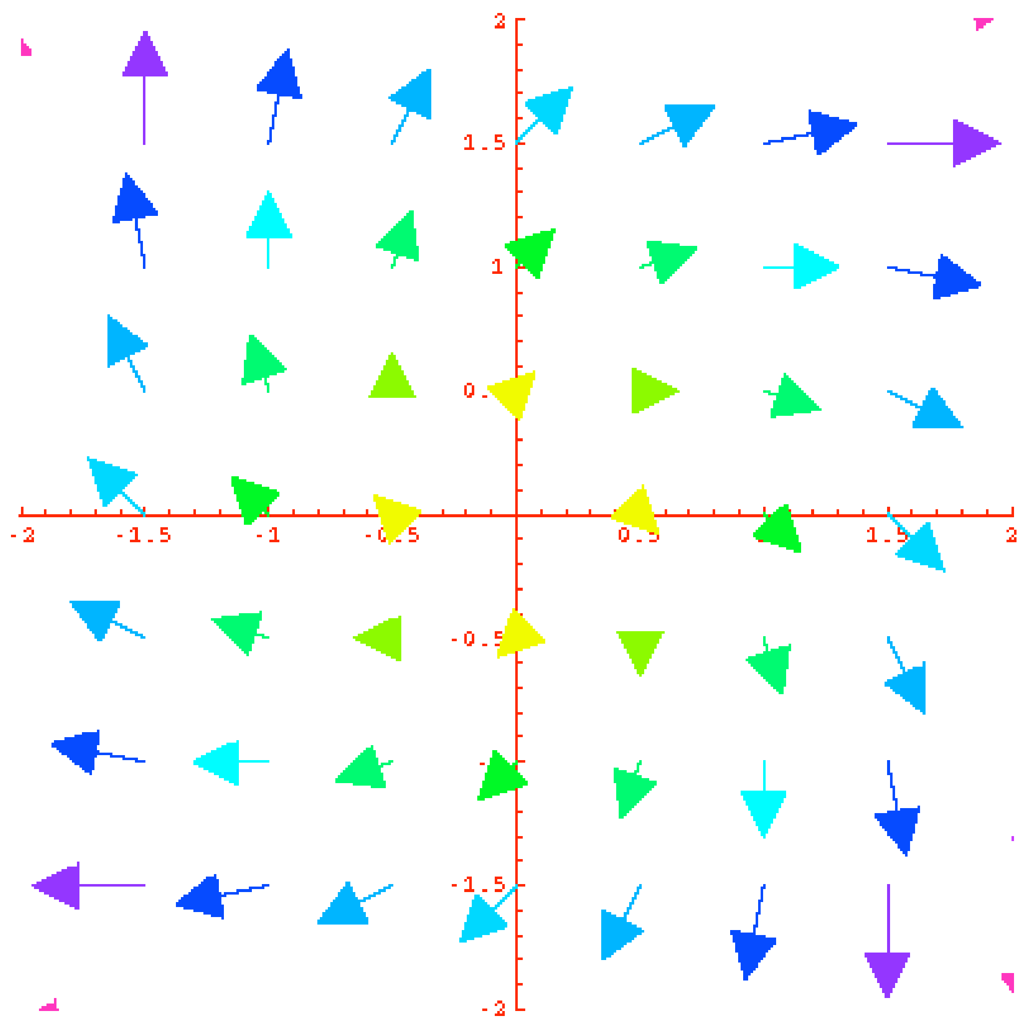


Complex eigenvalues

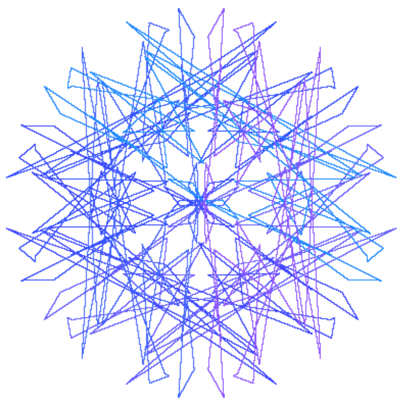


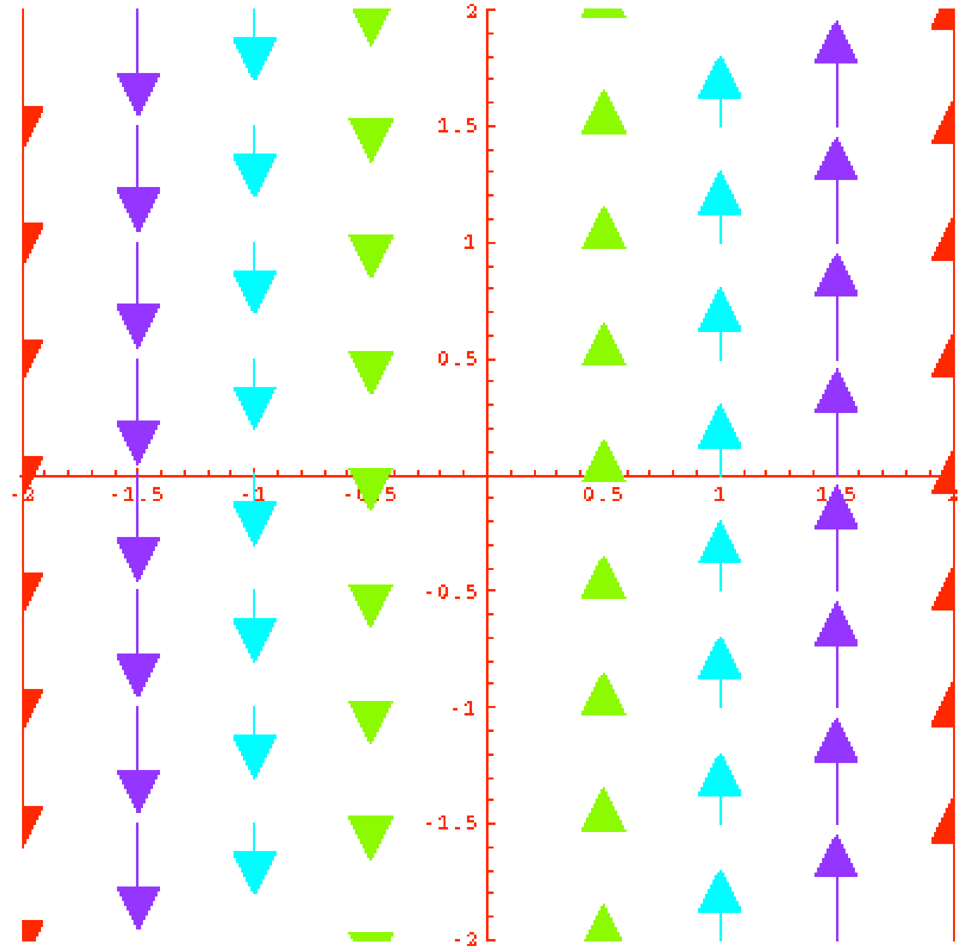


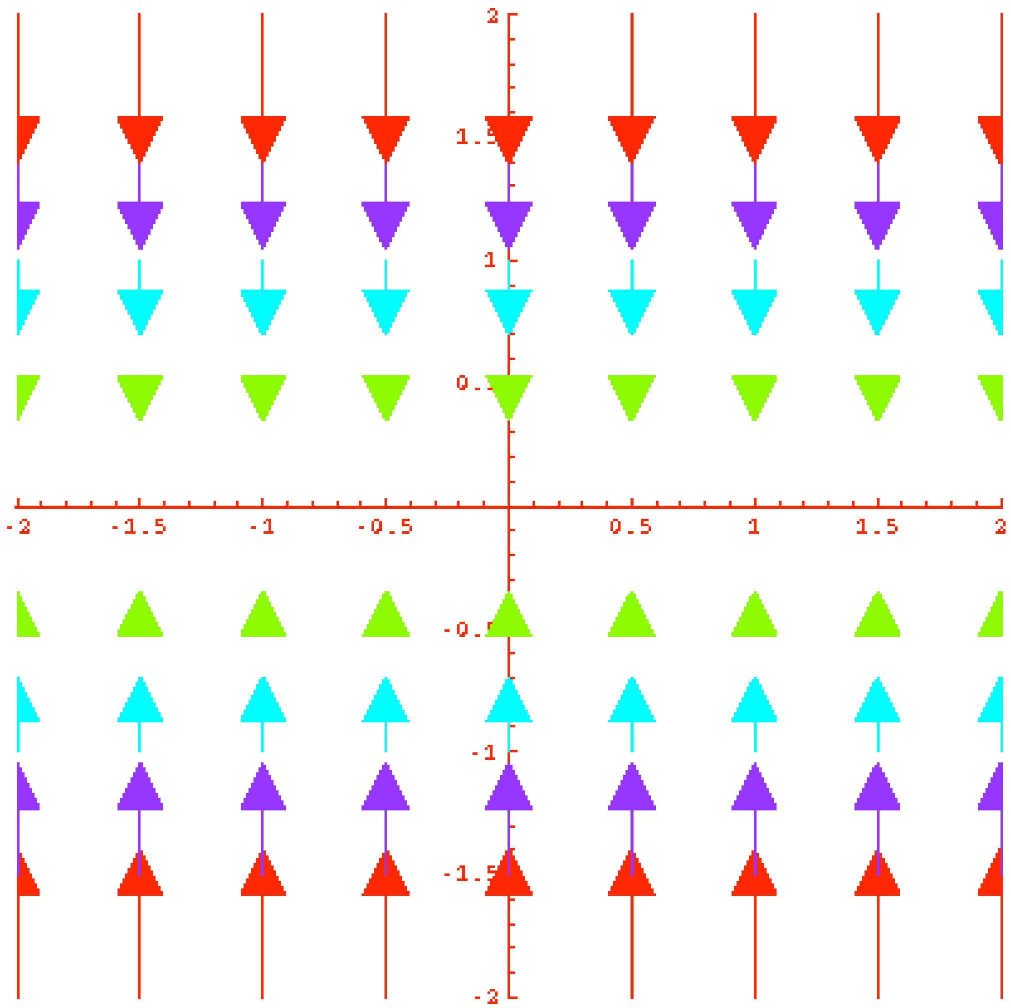


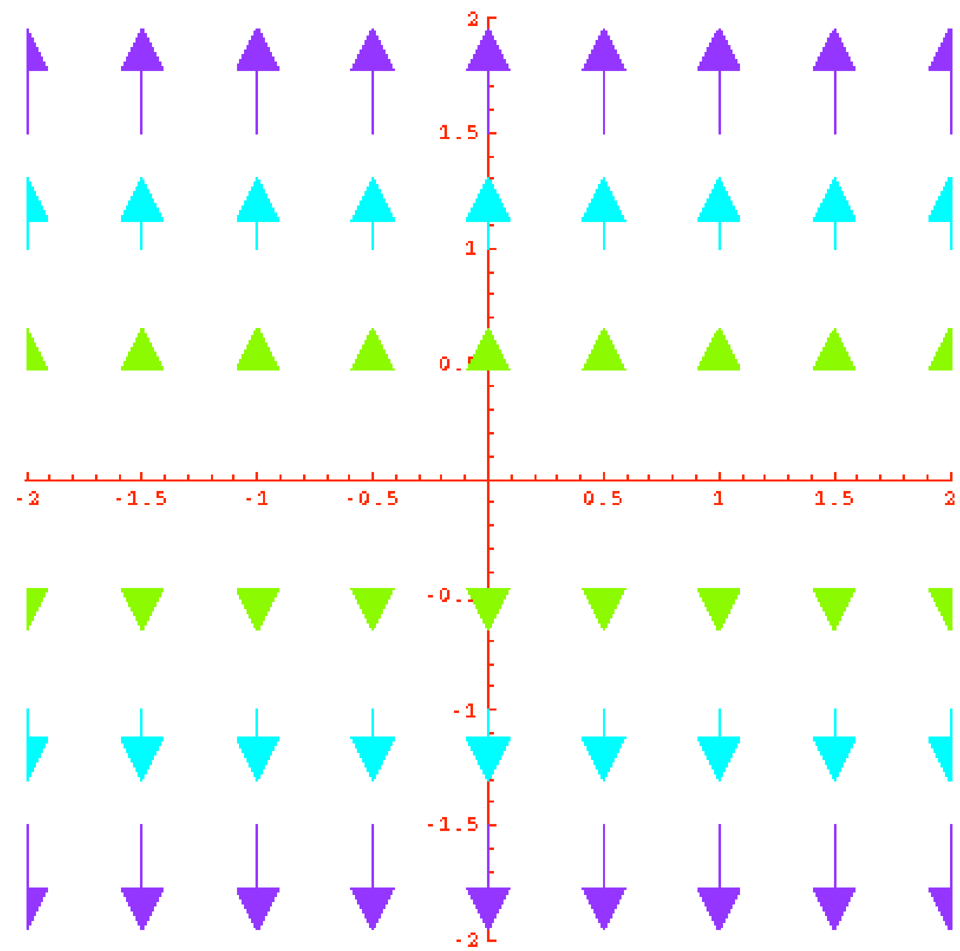


Some zero eigenvalue

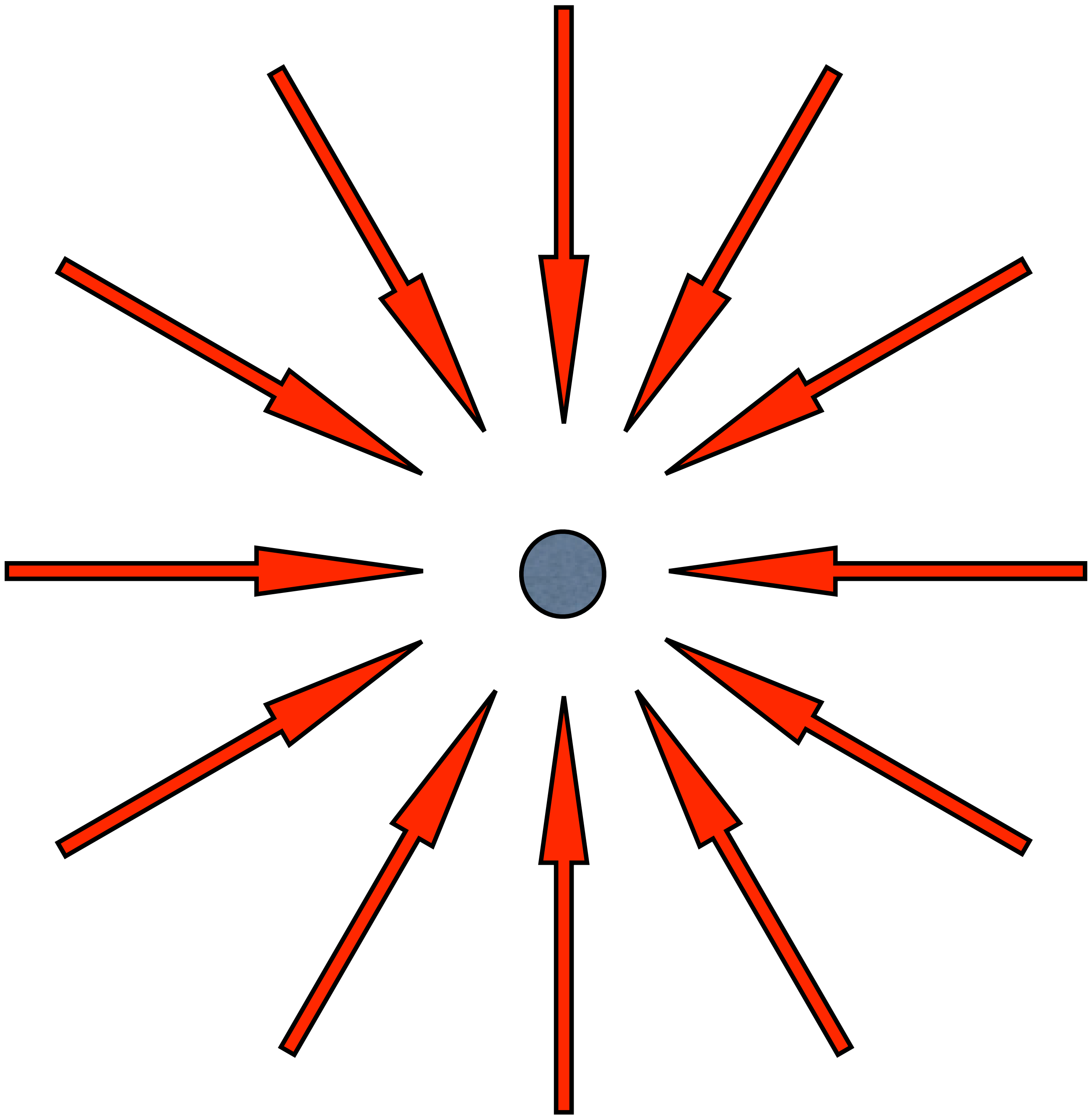


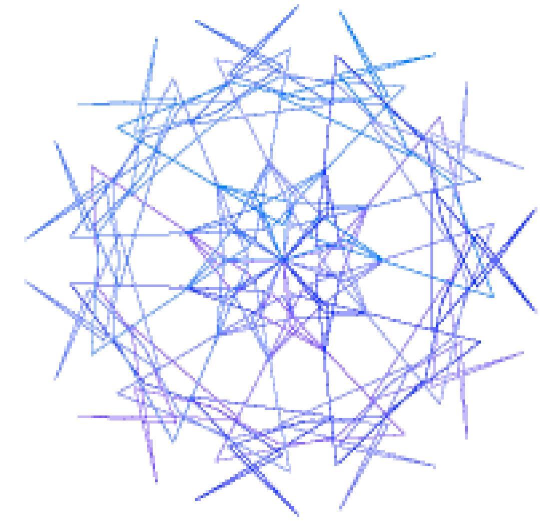




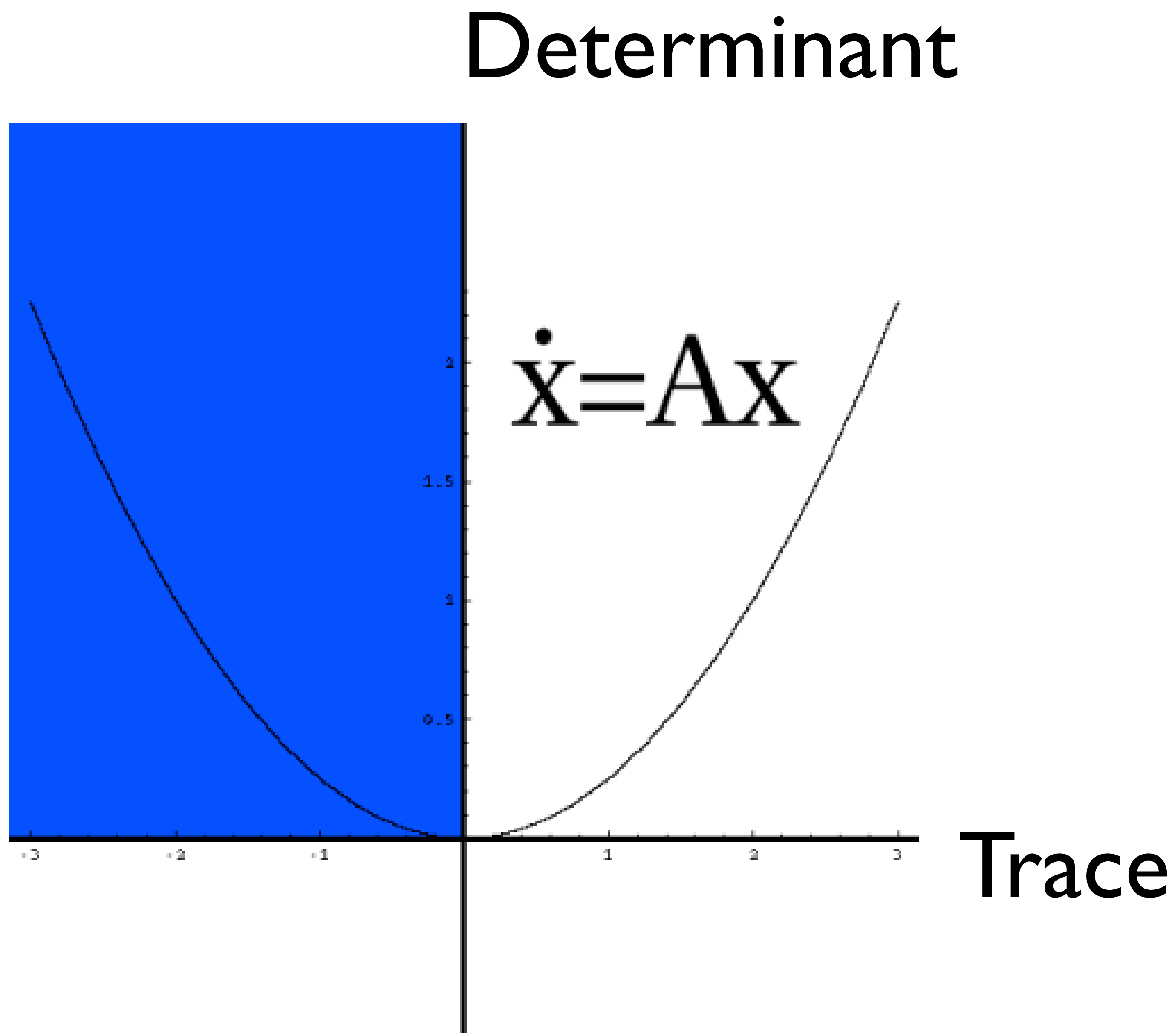


Asymptotic Stability





Stability for differential equation



Stability Examples

Example Blackboard

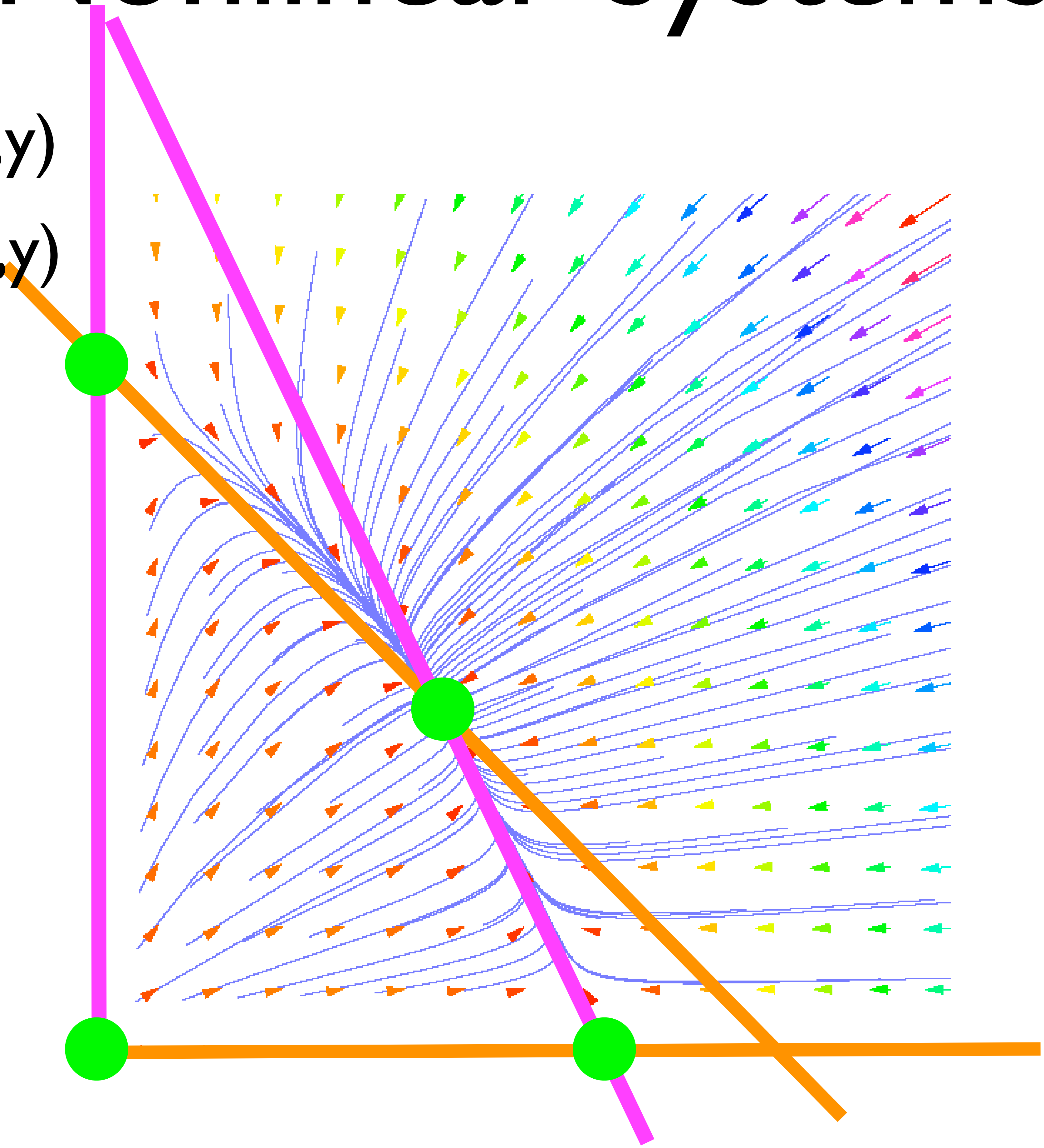
III) Nonlinear differential equation

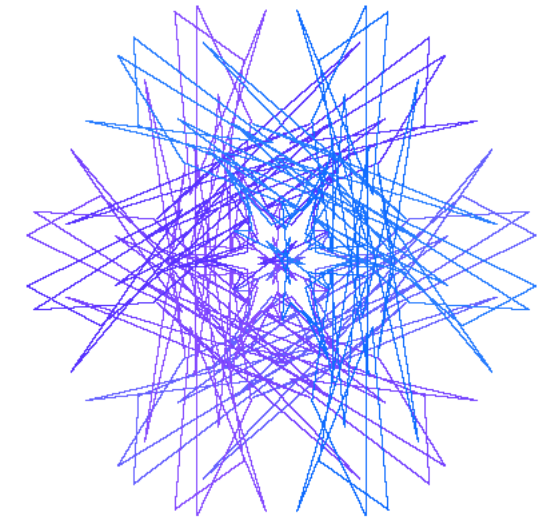
- Equilibrium points
- Nullclines
- Nature of equilibrium points
- Understanding phase space

Nonlinear Systems

$\dot{x}=f(x,y)$

$\dot{y}=g(x,y)$

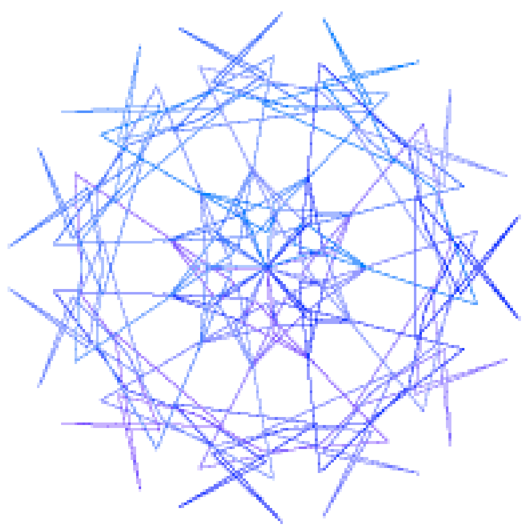


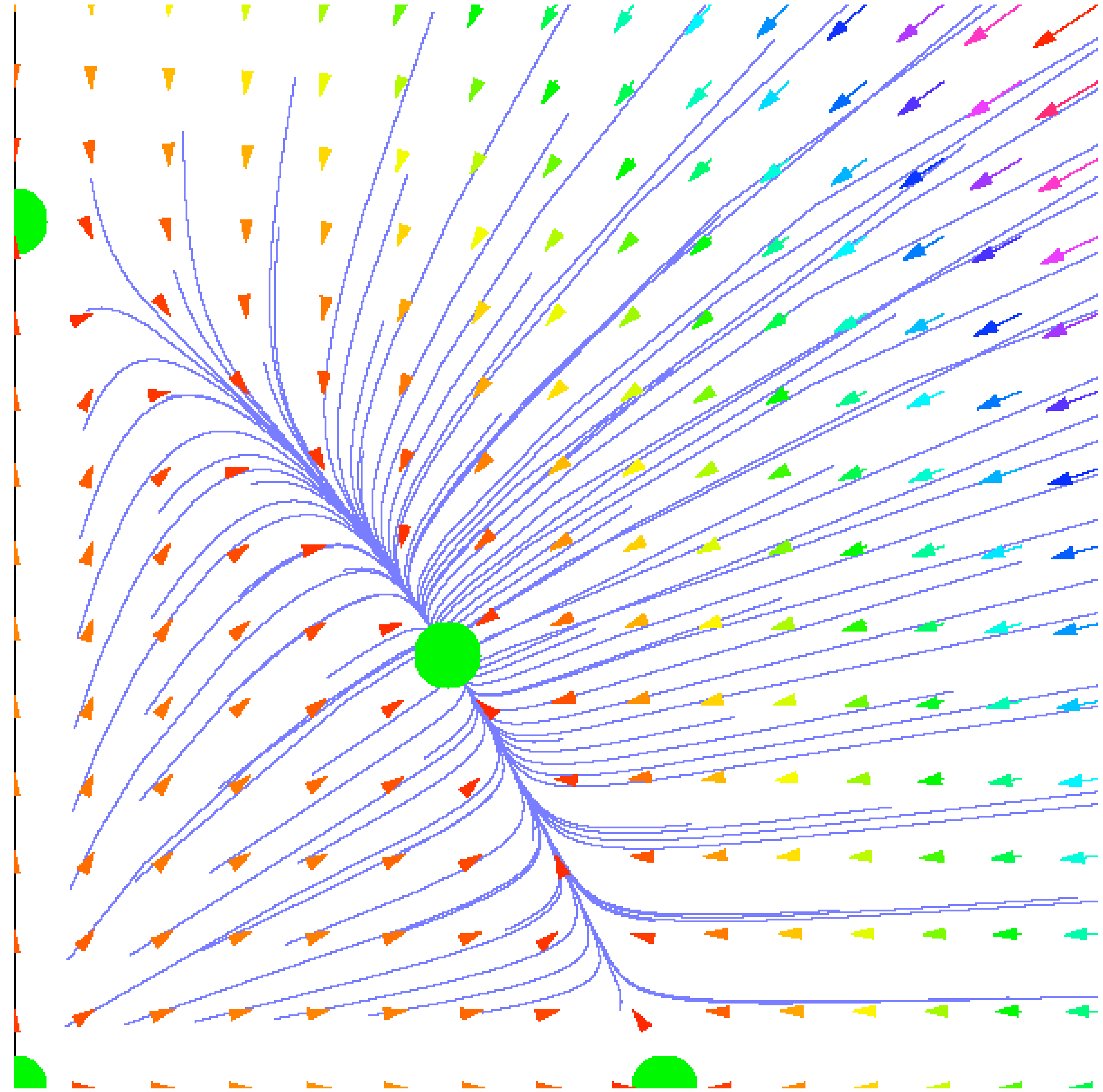


Null-clines

Equi-librium-points

Jacobean matrix



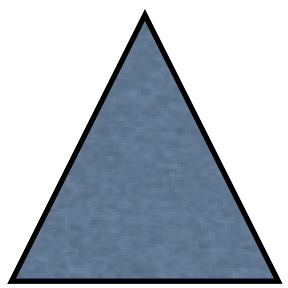


$$T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} f(x,y) \\ g(x,y) \end{bmatrix}$$

$$A\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} f_x(x,y) & f_y(x,y) \\ g_x(x,y) & g_y(x,y) \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

Intermezzo: pendulum

Example



Example Blackboard

IV) Higher Order Differential Equations

- Solving initial value problem p(D)f=g
- Homogenous problem p(D)f = 0
- Finding special solution to inhomogenous problem.

Higher Order ODE's

$$\frac{d}{dt} x(t) = Ax(t)$$

Important special case, which can be reduced to the above matrix notation are nonhomogenous higher order differential equations:

$$p(D) f(x) = g(x)$$

Initial Value Problem

Problem:

$$f'' + 5f = 2\sin(3x)$$
$$f(0)=1,\ f'(0)=2$$

Homogenous

$$f'' + 5f = 0$$

$$\lambda^2 + 5\lambda = 0$$
$$\lambda = i\sqrt{5}\quad -i\sqrt{5}$$

$$f(x) = a\sin(\sqrt{5}\,x) + b\cos(\sqrt{5}\,x)$$

Inhomogeneous

$$f'' + 5f = 2\sin(3x)$$

$$f(x) = c\sin(3x)$$
$$f'' + 5f = c + 5c = 2$$
so $c = -1/2$

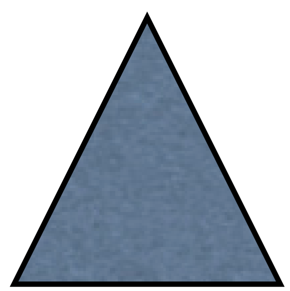
$$f(x) = -\sin(3x)/2$$

Initial conditions $f(0)=1$ implies $b=1$

$f'(0)=1$ implies $a = -3/(2\sqrt{5})$

Solution: $f(x) = -3/(2\sqrt{5})\sin(\sqrt{5}x) + \cos(\sqrt{5}x) - \sin(3x)/2$

Example

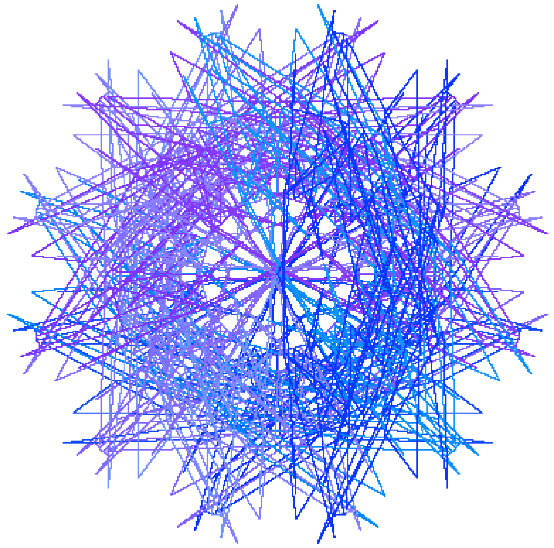


Example Blackboard

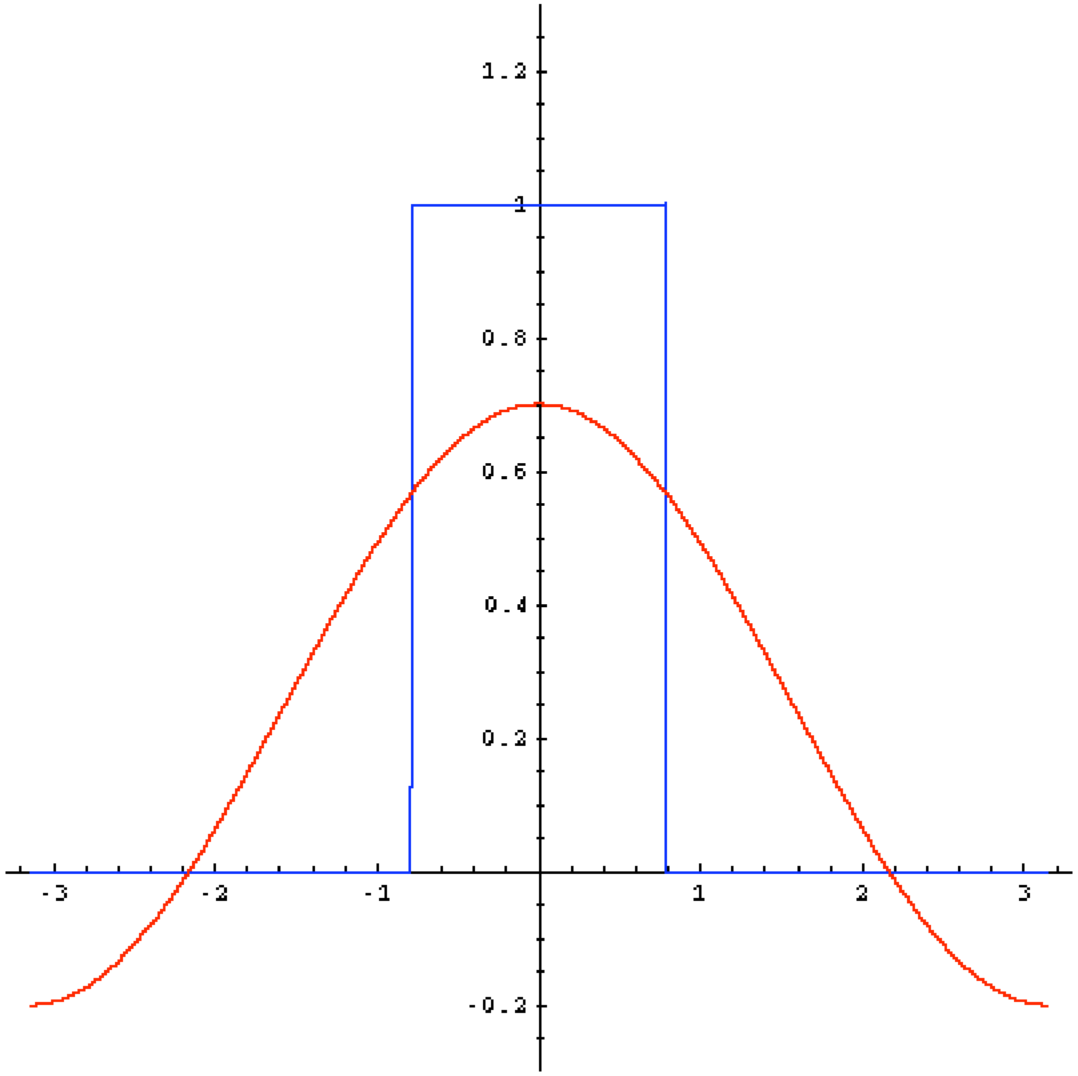
V) Fourier analysis

- Fourier Series
- Symmetry
- Parceval

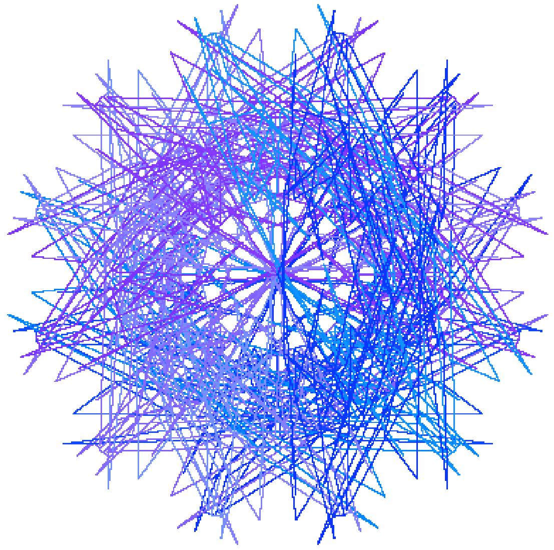
Fourier series



$$f(x) = \frac{a_0}{\sqrt{2}} + \sum_{n \geq 1} a_n \cos(nx) + b_n \sin(nx)$$



Fourier coefficients

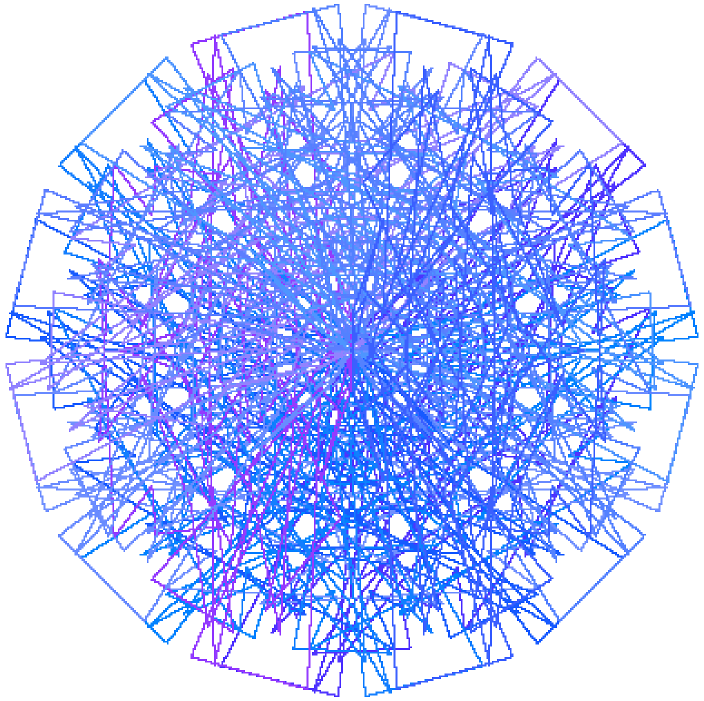


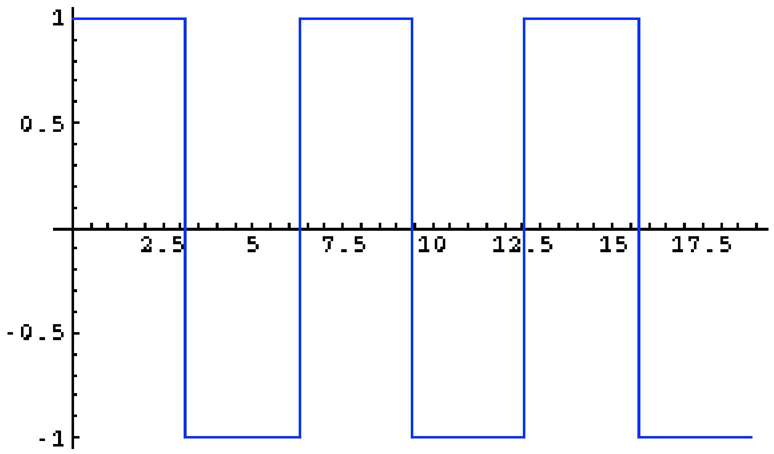
$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\frac{1}{\sqrt{2}}\, dx$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)\, dx$$

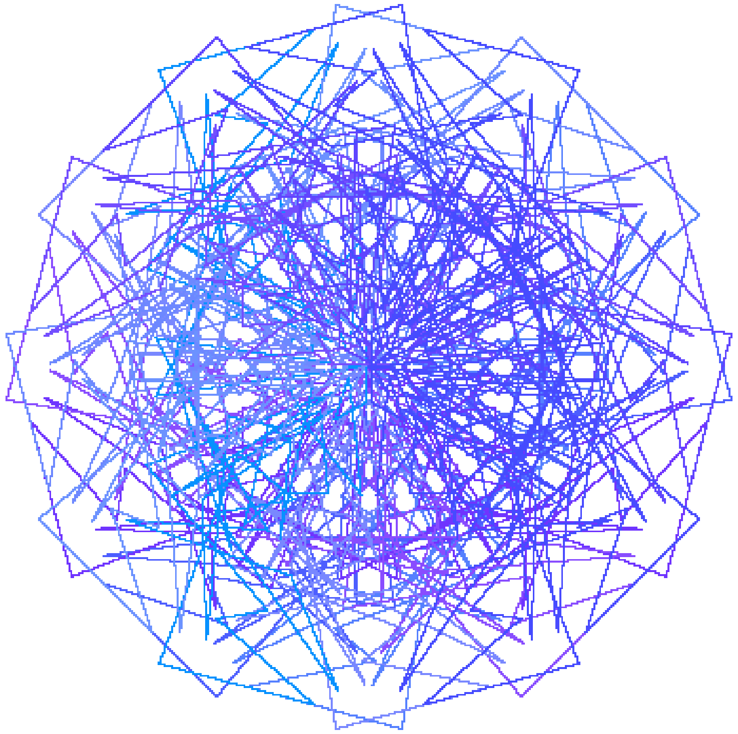
$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)\, dx$$

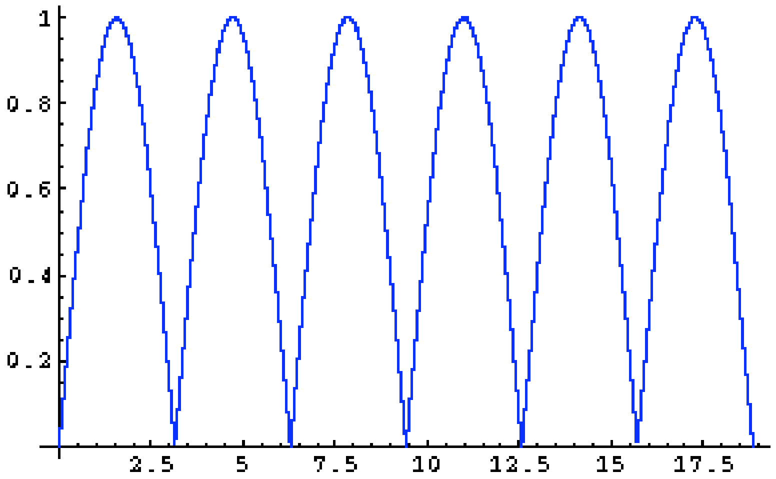
Sound synthesis I



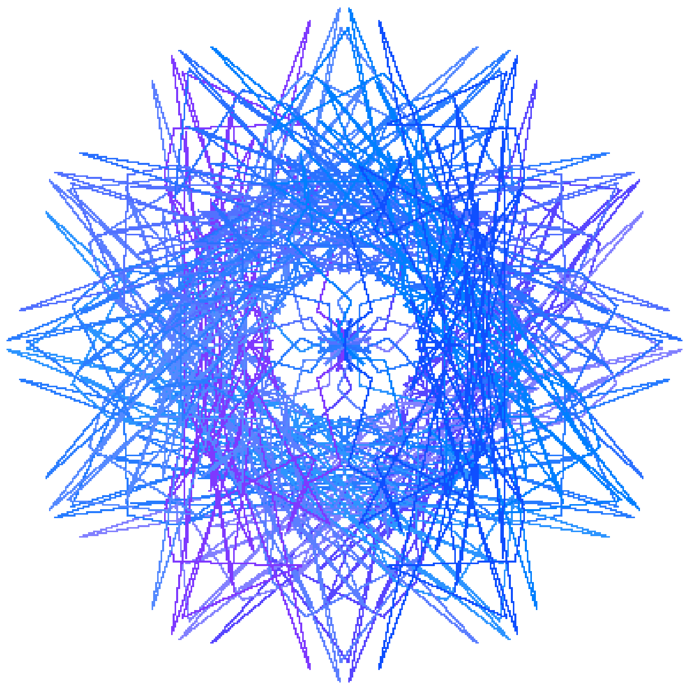


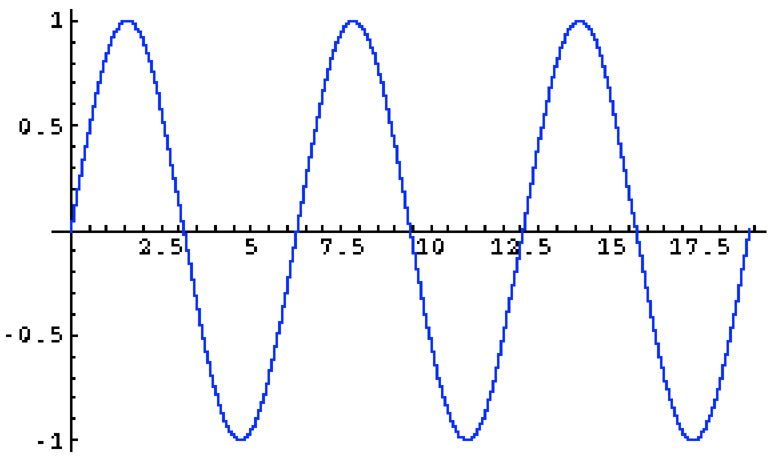
Sound synthesis II



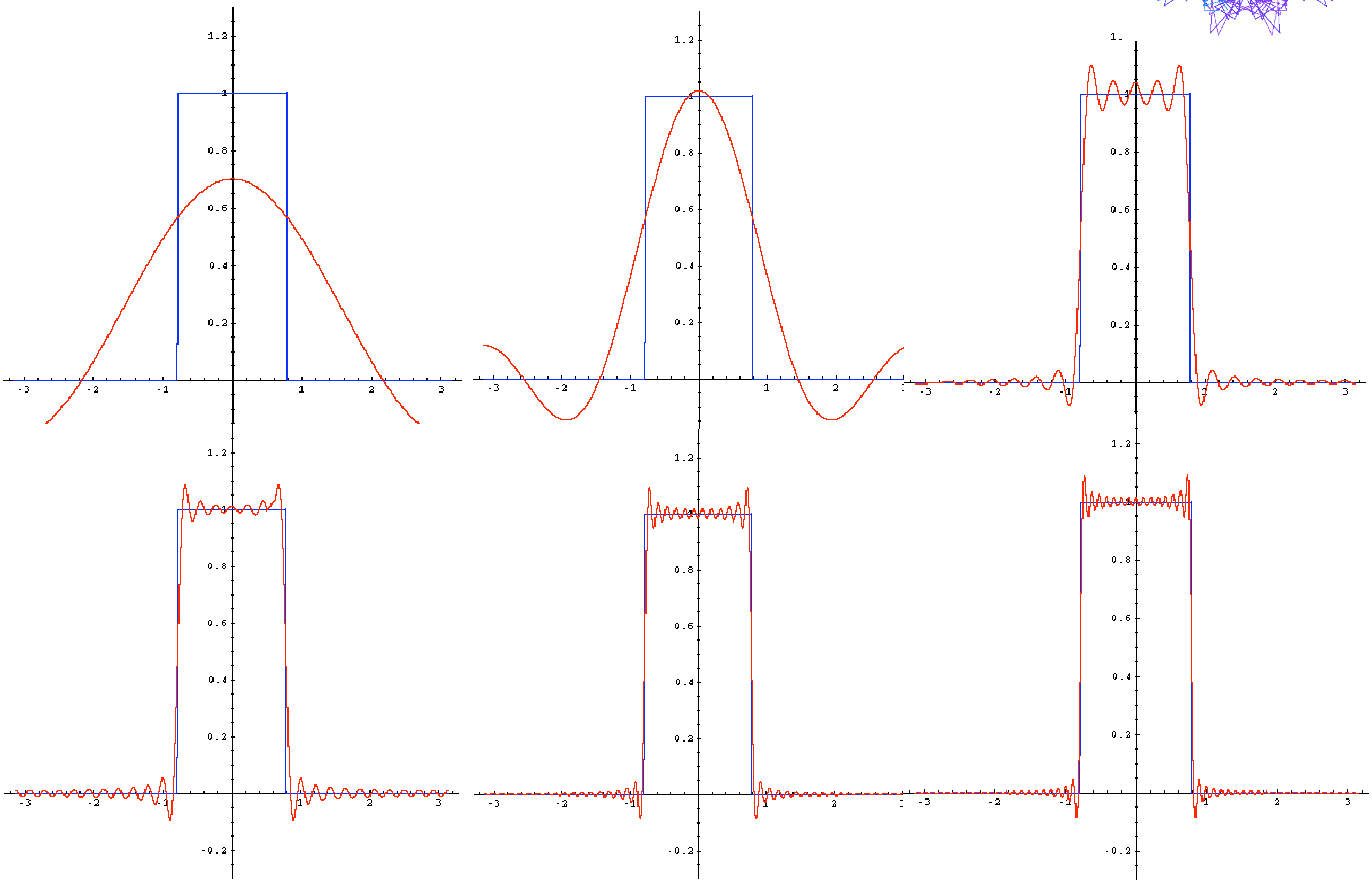


Sound synthesis III

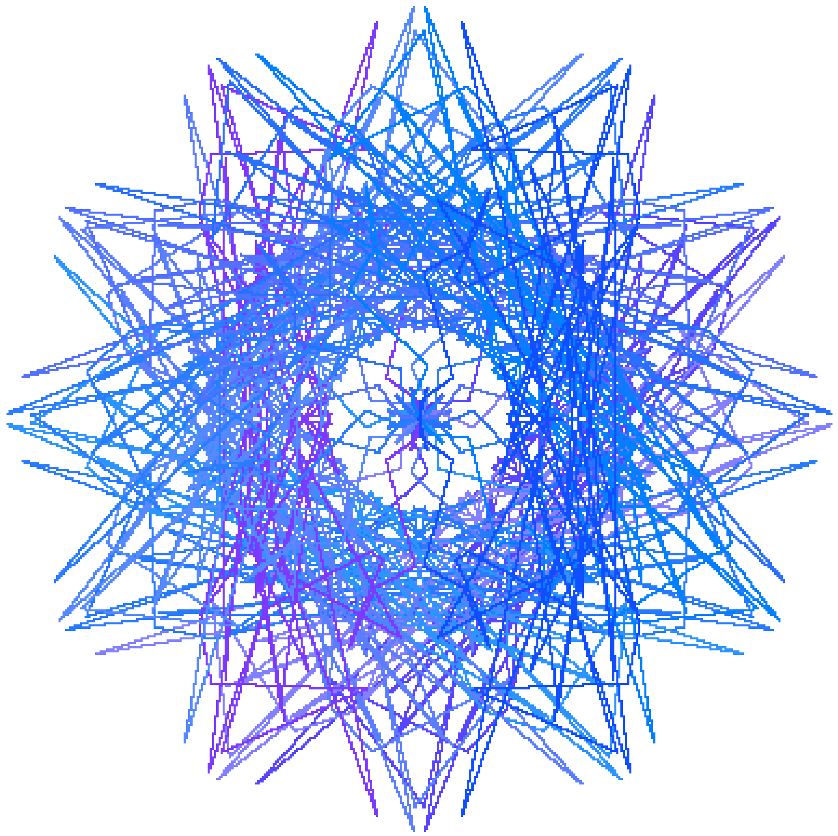


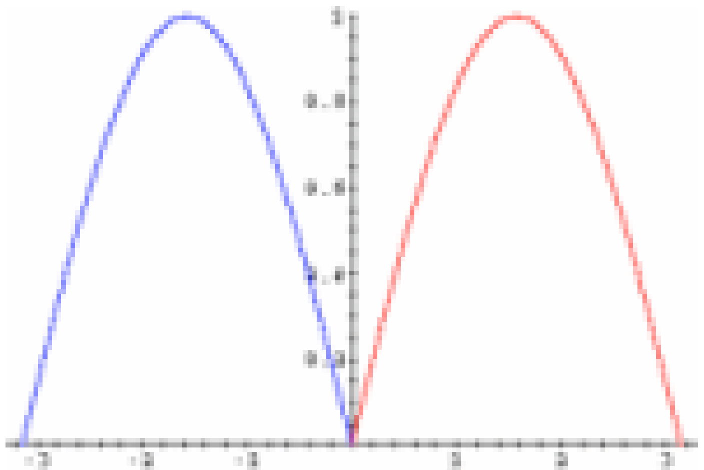


Fourier approximation

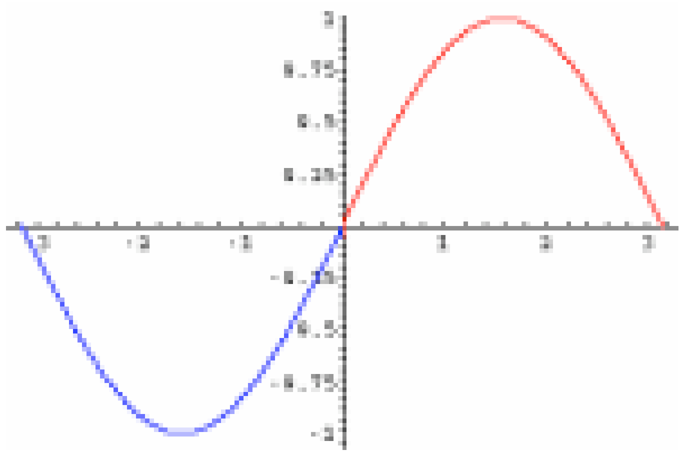


Even and Odd





cos-series



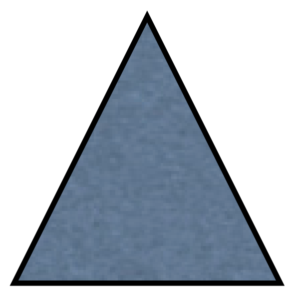
sin-series

Parseval

$$f(x) = \frac{a_0}{\sqrt{2}} + \sum_{n=1}^{\infty} a_n \cos(nx) + b_n \sin(nx)$$

$$\frac{a_0^2}{2} + \sum_{n=1}^{\infty} a_n^2 + b_n^2 = ||f||^2$$

Example

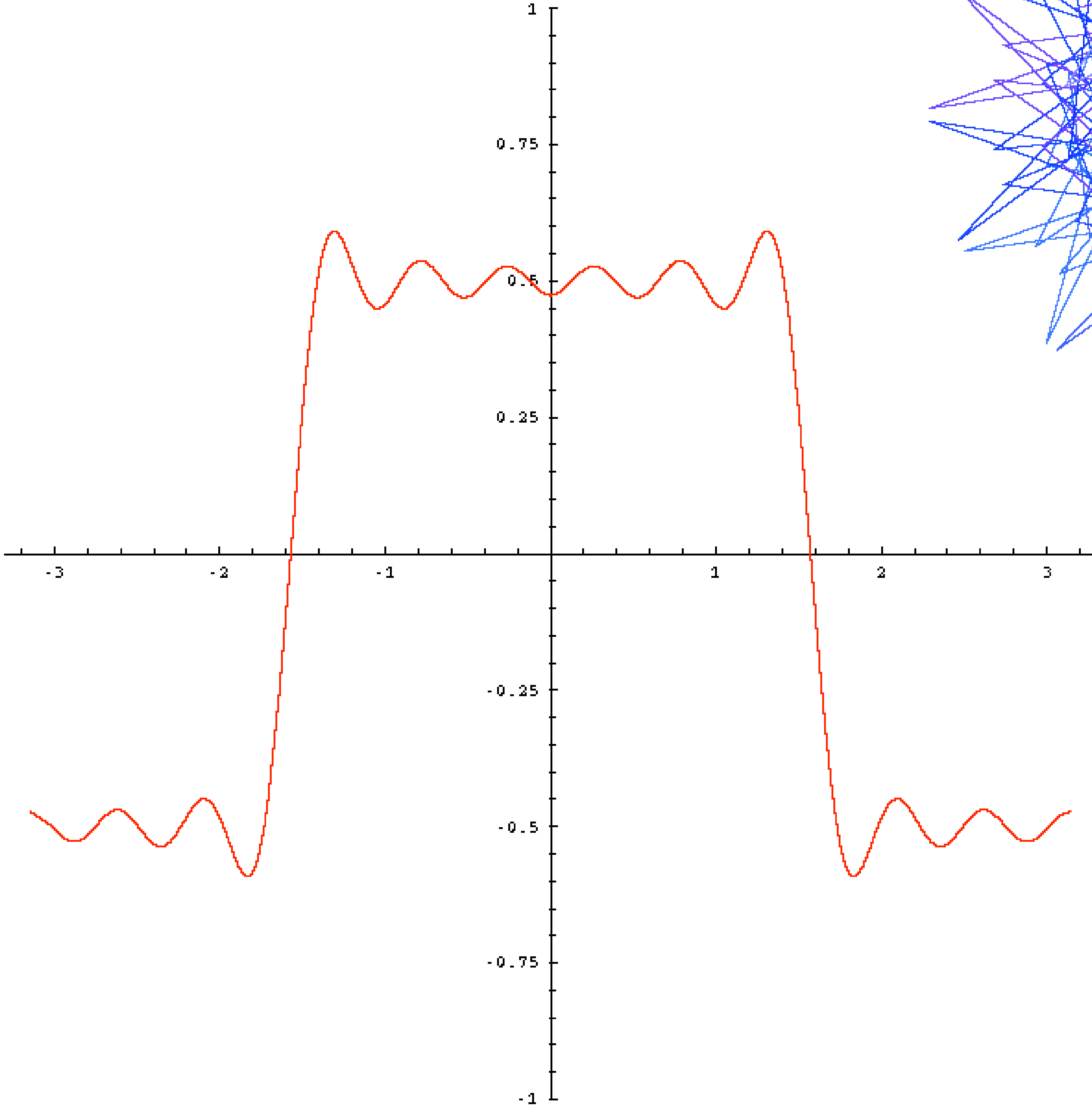


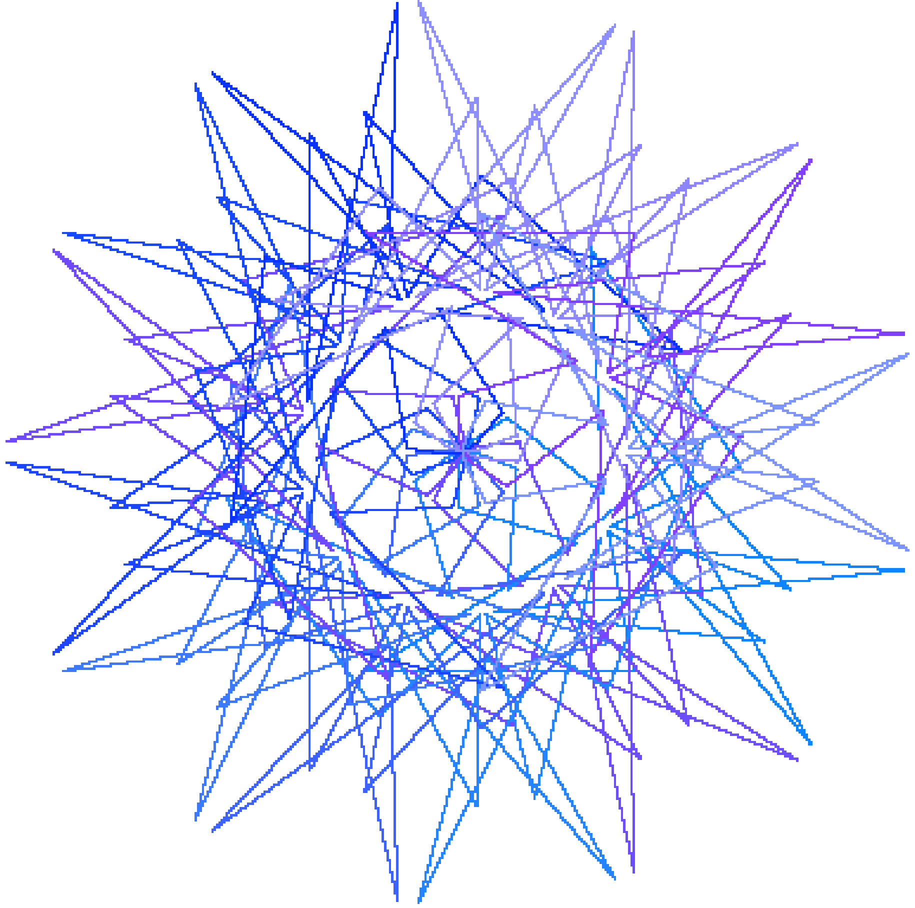
Example Blackboard

VI) Partial Differential equations

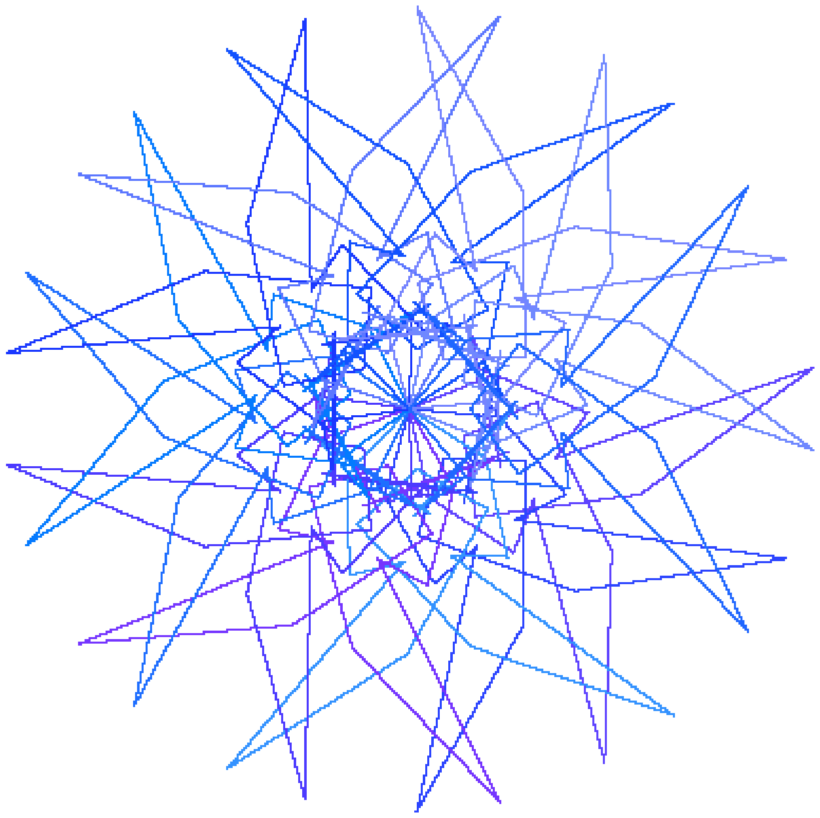
- Heat equation
- Wave equation
- Laplace equation (not covered)

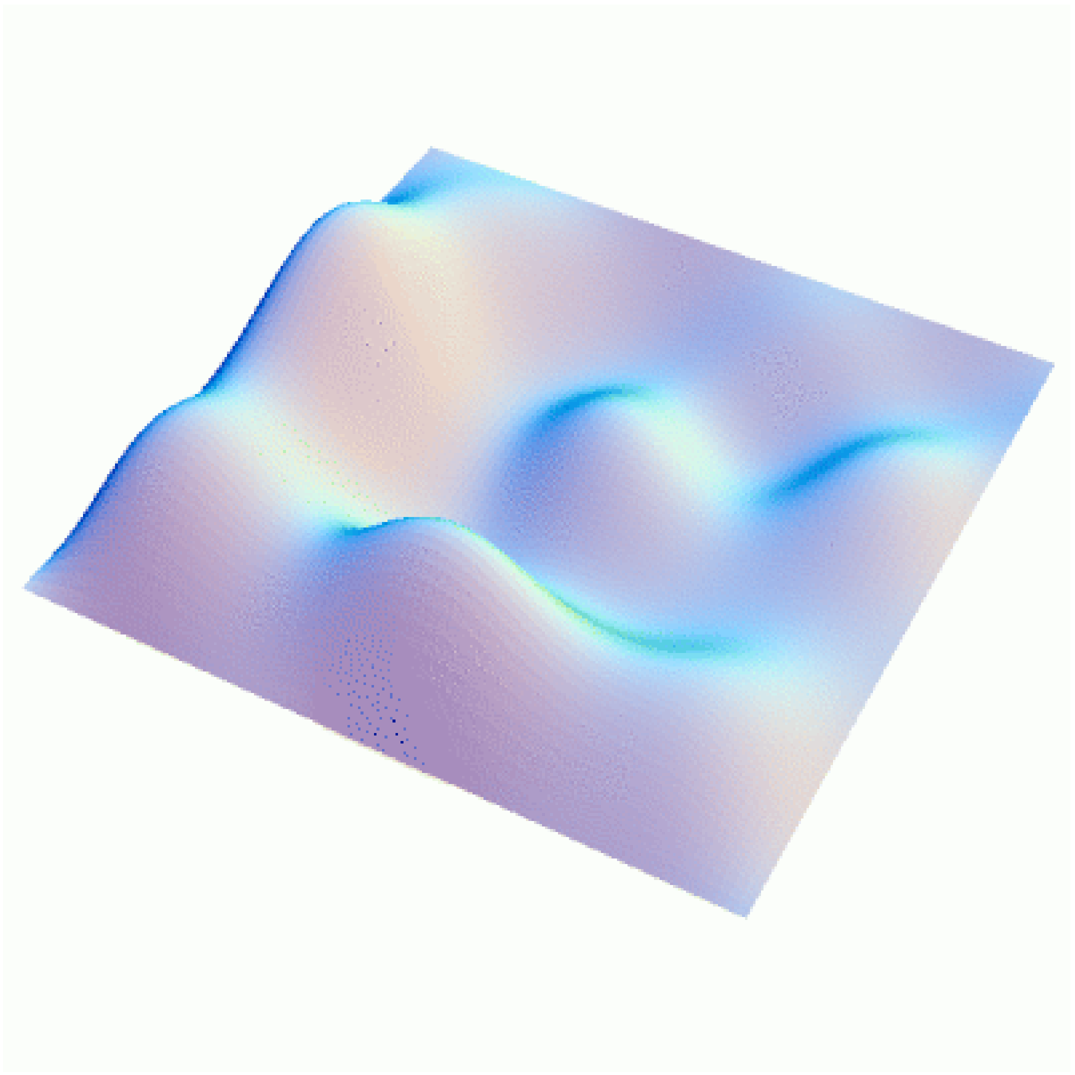
Heat Evolution

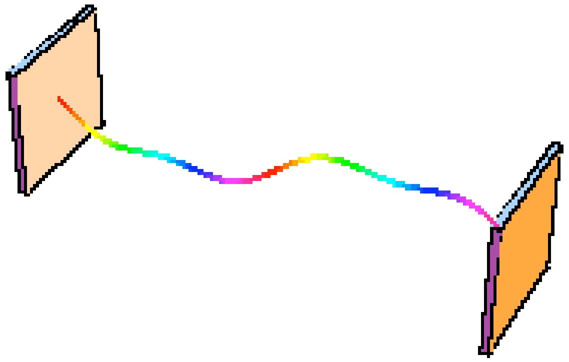




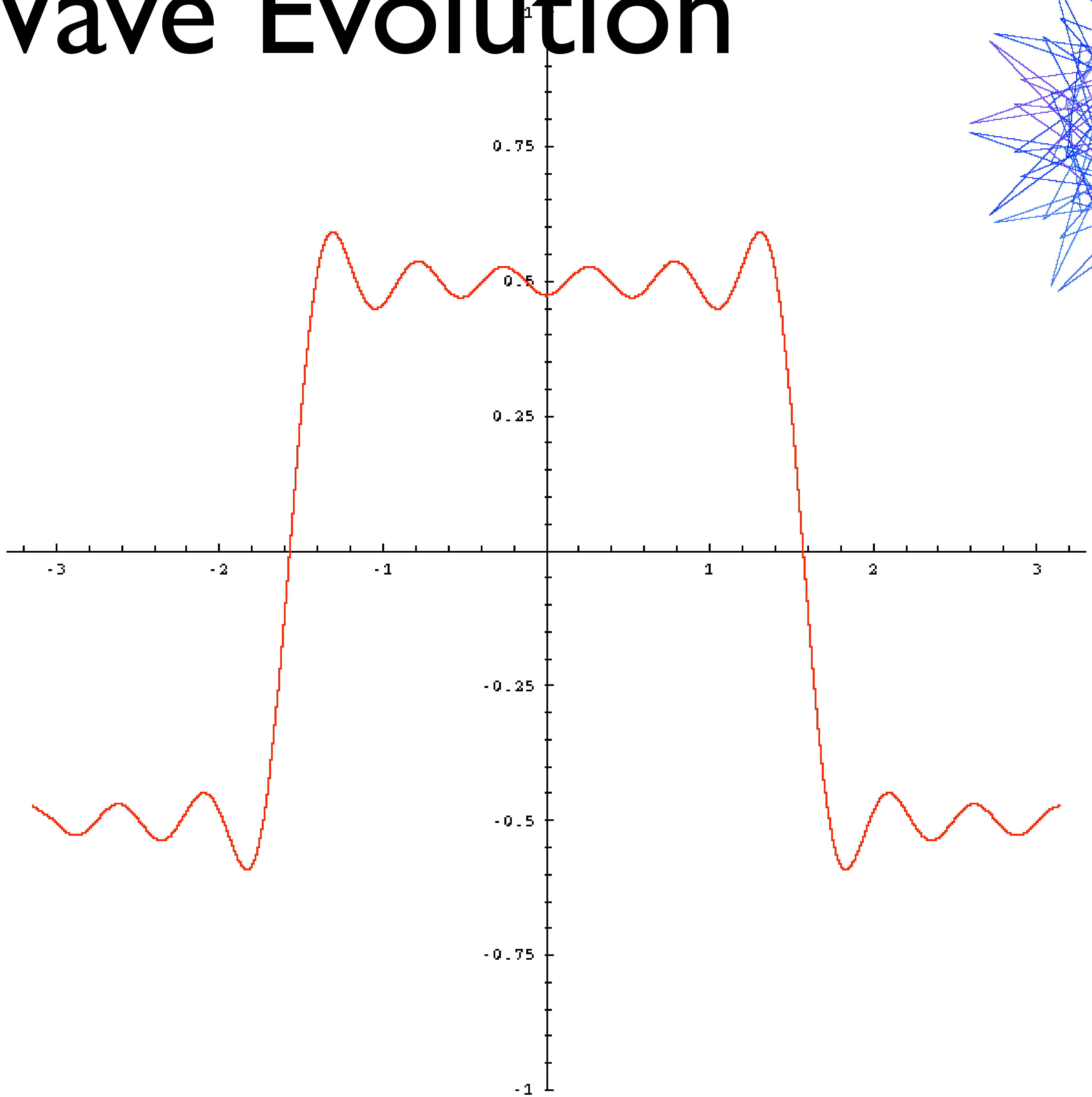
Waves

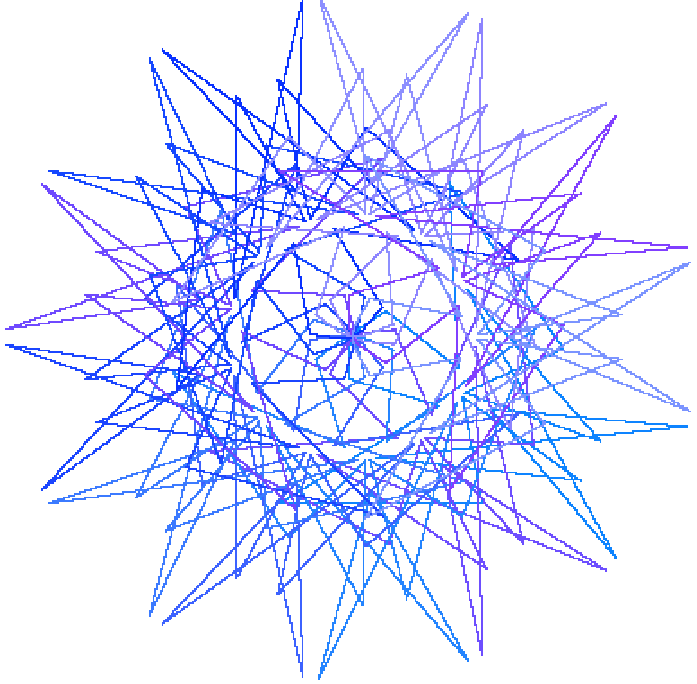




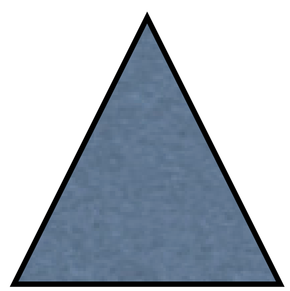


Wave Evolution





Example



Example Blackboard

Pro Memoria

- Exam is on January 27
- Review old exams and homework
- Do the practice exam