

RANDOM VARIABLES. In probability theory, functions are called **random variables**. They form a linear space.  $E[f] = \int f dx$  is called the **expectation** of a random variable  $f$ ,  $Var[f] = E[(f - E[f])^2]$  is called the **variance** of  $f$ ,  $Cov[f, g] = E[(f - E[f])(g - E[g])]$  is the **covariance** of  $f$  and  $g$ . One can think of  $Cov[f, g]$  as the inner product of  $f - E[f], g - E[g]$ .

LINEAR REGRESSION. Given data points  $(x_1, y_1), \dots, (x_n, y_n)$  we can find the line  $y = ax + b$  which best fits these points. We had the formula  $\begin{bmatrix} a \\ b \end{bmatrix} = (A^T A)^{-1} A^T \vec{b}$ , where  $\vec{b} = \begin{bmatrix} y_1 \\ \dots \\ y_n \end{bmatrix}$  and  $A = \begin{bmatrix} x_1 & 1 \\ x_2 & 1 \\ \dots & \dots \\ x_n & 1 \end{bmatrix}$ . Now  $A^T A =$