MATH 21B: LINEAR ALGEBRA
FALL 2005

Welcome to Linear Algebra! Life is multidimensional, going up to eleven dimensions according to some theoretical physicists, and there is more data in the world today than anyone can comprehend. Linear Algebra is the course that will give you the tools to handle all these extra dimensions and loads of data through matrix manipulations, the study of dynamical systems, an understanding of eigenvalues and eigenvectors as well as of differential equations. You can take this knowledge to many fields, from economics to epidemiology, from physics to the social sciences, and of course it would be crucial if you decide to continue in mathematics as well.

Enjoy your exploration of the landscape of Linear Algebra! This subject is well-known for being much more logically cohesive and self-contained than many subfields of mathematics including calculus, and yet it is astounding how many applications it has and how much it expands one's understanding of the world. Our crucial notion, the Matrix, will prove to be more than simply a set of numbers organized in rows and columns. Its name comes from the Latin word for *womb*, signifying its versatility in modelling the world surrounding us. Even though it is not an all-encompassing piece of software feigning reality as in the Hollywood movie by the same name, in the hands of a skilled user, it can come pretty close. So follow the white rabbit!

Introductory Meeting: Mon, Sept 19 at 8:30am in Sci Center A.

Note that this meeting is **required** for anyone wishing to enroll in Math 21b.

Instructors: Andreea Nicoara (course head)
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Office hours: Mon 12-1pm, Fri 2-3pm

Christopher Phillips
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Course assistants: Petko Peev, peev@fas.harvard.edu
(Andreea Nicoara's section)
Philip Powell, ppowell@fas.harvard.edu
(Chris Phillips' section)

Lectures: Mon, Wed, Fri 10-11 in location SC B-10 (Chris Phillips' section)
Mon, Wed, Fri 11-12 in location SC 110 (Andreea Nicoara's section)

Problem sessions: Mon 5-6:30pm in SC 310
Philip Powell (Chris Phillips' section)
Tue 6:30-8pm in SC 310
Petko Peev (Andreea Nicoara's section)

Course web site: http://www.courses.fas.harvard.edu/∼math21b/

Textbook: Otto Bretscher's *Linear Algebra with Applications*, third edition, published by Prentice Hall and available at the Harvard Coop. Please make sure to buy the third edition since there are differences in content among the different editions. The book was written by Otto Bretscher, who has been teaching this very course at Harvard for many years. Not only is this the very best textbook available, but it has been especially tailored to fit the Harvard schedule and meet the needs of Harvard students. We will cover roughly a section a day, except for the very last topics on differential equations for which handouts will be posted on the course web site. Do make sure to *read* the relevant section(s) before each lecture and again after. This will allow you to learn the material much more thoroughly.

Prerequisites: Before beginning this class, you are expected to have had the equivalent of a second semester course in calculus such as Math 1b. In addition, it would be helpful to have taken multivariable calculus as well at the level of Math 21a, although this is not absolutely necessary.

Sectioning: After the introductory meeting, Math 21b is taught entirely in sections. To enroll in the course, you must section by computer starting Monday, September 19th, and **no later than 1:00pm on Wednesday, September 21st.** If you have an email account, log on to the Harvard computer system, then type

ssh section@ulam.fas.harvard.edu

instead of "pine", and follow the instructions. For additional help, please go to the Math Department's home page,

$$http://www.math.harvard.edu$$

and click on the sectioning link in the upper right corner of the page. If you are having problems sectioning, please contact Susan Milano via email at milano@math.harvard.edu no later than 1:00pm on Wednesday. Your section assignment will be sent to you via email by 5:00pm on Friday, September 23rd.

Homework: Homework problems will be assigned at each meeting of your section, and they are due at the following meeting, unless explicitly stated otherwise. Late homework is discouraged. Should you have an emergency, please let your instructor know before the homework is due.

Feel free to form study groups with your fellow students in order to work on the problem sets, but make sure to write up the solutions *by yourself*.

The homework assignments will be posted on the Math 21b web site. The answers to the homework assignments will appear after the due date on the web site as well. Moreover, selected problems from the homework will be discussed in the problem sessions.

Often harder extra credit problems will be assigned. These will be marked by $(*)$. Note also that your three *lowest* homework grades will be dropped when your homework average is computed.

Math Question Center: In addition to class, problem sessions and office hours, the Math Department runs a Math Question Center in Loker on Sunday through Thursday evenings from 8pm to 10pm. The Question Center is staffed by Course Assistants as well as graduate students. This is a good place to meet with other students in your class to do homework. You should feel free to drop by any time you want a bit of help, or if you just want to solidify your basic math understanding by doing some review problems.

Technology: Graphing calculators and mathematical software programs such as *Matlab* and *Mathematica* are high-tech toys with which you can feel free to play as you are exploring the course material. Note, however, that *no calculators* will be allowed during exams. We will make sure to give you exam problems that test your mathematical understanding and not your computing prowess.

Exams: There will be one midterm and one final exam. The schedule is as follows:

Midterm Exam: Tues, Nov 1 from 7-9pm in Sci Center D

Final Exam: Sat, Jan 14 from at time TBA in location TBA

It is your responsibility to resolve any scheduling conflicts. Note that exams take precedence over any other obligation you might have. If there are indeed very serious grounds that warrant an exception, please let your instructor know as soon as possible.

Grading: Course grades will be computed roughly based on the following formula: homework (30%), midterm (30%), and final (40%).

Syllabus: The following schedule is a *tentative* one. The numbers refer to Bretscher's text.

Lecture 1: Monday 9/26 Chapter 1.1
Introduction to Linear Systems

Lecture 2: Wednesday 9/28 Chapter 1.2
Matrices, Vectors, and Gauss-Jordan Elimination

Lecture 3: Friday 9/30 Chapter 1.3
On the Solutions of Linear Systems; Matrix Algebra

Lecture 4: Monday 10/3 Chapter 2.1
Introduction to Linear Transformations and Their Inverses

Lecture 5: Wednesday 10/5 Chapter 2.2
Linear Transformations in Geometry

Lecture 6: Friday 10/7 Chapter 2.3
The Inverse of a Linear Transformation

Columbus Day: Monday 10/10 No class – university holiday

Lecture 7: Wednesday 10/12 Chapter 2.4
Matrix Products

Lecture 8: Friday 10/14 Chapter 3.1
Image and Kernel of a Linear Transformation

Lecture 9: Monday 10/17 Chapter 3.2
Subspaces of $\mathbb{R}^n$; Bases and Linear Independence

Lecture 10: Wednesday 10/19 Chapter 3.3
The Dimension of a Subspace of $\mathbb{R}^n$

Lecture 11: Friday 10/21 Chapter 3.4
Coordinates

Lecture 12: Monday 10/24 Chapter 5.1
Orthogonal Projections and Orthonormal Bases

Lecture 13: Wednesday 10/26 Chapter 5.2
Gram-Schmidt Process and QR Factorization

Lecture 14: Friday 10/28 Chapter 5.3
Orthogonal Transformation and Orthogonal Matrices

Lecture 15: Monday 10/31 Review for the midterm exam

Tuesday, November 1st: Midterm Exam from 7-9pm in Science Center D

Lecture 16: Wednesday 11/2 Chapter 5.4
Least Squares and Data Fitting

Lecture 17: Friday 11/4 Chapters 6.1 and 6.2
Introduction to Determinants and Properties of Determinants

Lecture 18: Monday 11/7 Chapter 7.1
Dynamical Systems and Eigenvectors: An Introductory Example

Lecture 19: Wednesday 11/9 Chapter 7.2
Finding the Eigenvalues of a Matrix

Veterans' Day: Friday 11/11 No class – university holiday

Lecture 20: Monday 11/14 Chapter 7.3
Finding the Eigenvectors of a Matrix

Lecture 21: Wednesday 11/16 Chapter 7.4
Diagonalization

Lecture 22: Friday 11/18 Chapter 7.5
Complex Eigenvalues

Lecture 23: Monday 11/21 Chapter 7.6
Stability

Lecture 24: Wednesday 11/23 Chapter 8.1
Symmetric Matrices

Thanksgiving Recess: Friday 11/25 No class – university holiday

Lecture 25: Monday 11/28 Chapter 9.1
An Introduction to Continuous Dynamical Systems

Lecture 26: Wednesday 11/30 Chapter 9.2
The Complex Case: Euler's Formula

Lecture 27: Friday 12/2 Otto Bretscher's Supplement
Nonlinear Systems

Lecture 28: Monday 12/5 Handout 10.1 and Chapter 9.3
Linear Differential Operators I

Lecture 29: Wednesday 12/7 Handout 10.1 and Chapter 9.3
Linear Differential Operators II

Lecture 30: Friday 12/9 Handout 10.2 and Chapter 5.5
Fourier Series

Lecture 31: Monday 12/12 Handout 10.2 and 10.3
Fourier Series Continued and the Heat Equation

Lecture 32: Wednesday 12/14 Handout 10.3 and 10.4
The Heat Equation Continued and Laplace's Equation

Lecture 33: Friday 12/16 Handout 10.4
Finish Laplace's Equation and Introduce the Wave Equation

Lecture 34: Monday 12/19 Review

Wednesday, December 21st – Monday, January 2nd: Winter Recess
Tuesday, January 3rd – Friday, January 13th: Reading Period
Review Sessions for the final exam TBA

Saturday, January 14th: Final Exam at time TBA in location TBA